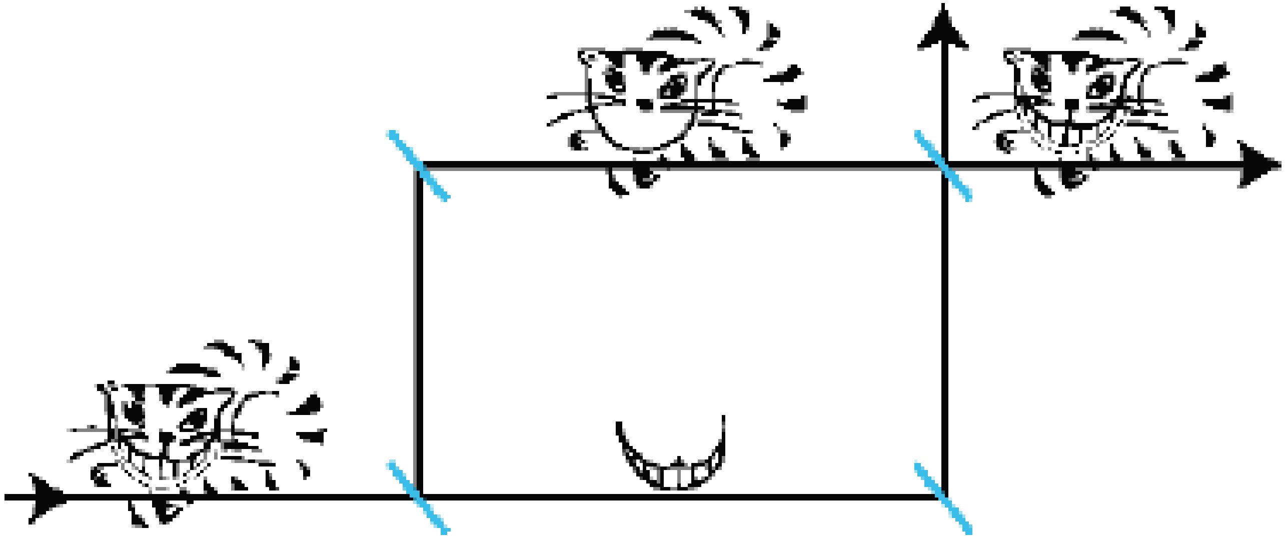
# Mesures Faibles et Chat du Cheshire

**??????**



Exposé de Daniel VAN LABEKE
Epiphymaths jeudi 28 mai 2015

VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

## How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.

PHYSICAL REVIEW D VOLUME 40, NUMBER 6 15 SEPTEMBER 1989

## The sense in which a "weak measurement" of a spin-$\frac{1}{2}$ particle's spin component yields a value 100

I. M. Duck and P. M. Stevenson

We give a critical discussion of a recent Letter of Aharonov, Albert, and Vaidman. Although their work contains several flaws, their main point is valid: namely, that there is a sense in which a certain "weak measurement" procedure yields values outside the eigenvalue spectrum. Our analysis requires no approximations and helps to clarify the physics behind the effect. We describe an optical analog of the experiment and discuss the conditions necessary to realize the effect experimentally.

**Overview of Weak Values and Enhancement of Small Signals in Physics and Chemistry**

Joseph S. Choi[1,*]

The AAV theory as originally written had many errors,and its interpretations have had confusing conclusions (such as complex weak values, negative probabilities, su-perluminal propagation, and time travel) that are still highly debated today [6, 8]. Duck, Stevenson, and Sudarshan (DSS) provided a nice paper a year after, to cor-rectly state the theory, while confirming the conclusions of AAV, namely that a measurement value can lie out-side of the eigenvalue spectrum with the proposed weak value procedure

# Mesure faible

The quantum system, whose observable $\hat{A}$ is to be measured, is coupled to a measuring device by a coupling Hamiltonian

$$\hat{H} = -g(t)\hat{q}\,\hat{A}$$

$$|\Phi_{in}\rangle = \begin{cases} \int dq\, \phi_{in}(q)|q\rangle & (q \text{ representation}), \\ \int dp\, \tilde{\phi}_{in}(p)|p\rangle & (p \text{ representation}), \end{cases}$$

$$\phi_{in}(q) \equiv \langle q|\Phi_{in}\rangle = \exp\left[-\frac{q^2}{4\Delta^2}\right],$$

$$\tilde{\phi}_{in}(p) \equiv \langle p|\Phi_{in}\rangle = \exp(-\Delta^2 p^2),$$

$$\Delta q \equiv \Delta, \quad \Delta p = 1/(2\Delta),$$

$$|\Psi_{in}\rangle = \sum_n \alpha_n |A = a_n\rangle$$

$$\exp\left[-i\int \hat{H}\, dt\right]|\Psi_{in}\rangle|\Phi_{in}\rangle = \sum_n \alpha_n \int dq\, e^{iqa_n} \exp\left[\frac{-q^2}{4\Delta^2}\right]|A = a_n\rangle|q\rangle$$

$$\sum_n \alpha_n \int dp\, \exp[-\Delta^2(p - a_n)^2]|A = a_n\rangle|p\rangle$$

# Mesure faible (2)

$$\exp\left[-i\int \hat{H}\,dt\right]|\Psi_{\text{in}}\rangle|\Phi_{\text{in}}\rangle = \sum_n \alpha_n \int dp\,\exp[-\Delta^2(p-a_n)^2]\,|A=a_n\rangle|p\rangle$$

Thus, if $\Delta p = 1/(2\Delta)$ is small compared to the spacing between the $a_n$'s, the measuring device is left in a state consisting of widely separated "spikes," each centered on one of the eigenvalues $a_n$. Hence, in the limit $\Delta p \to 0$, one has all the properties of an ideal measurement: (i) the measurement always produces one of the eigenvalues $a_n$; (ii) the probability of the outcome $a_n$ is $|\alpha_n|^2$; (iii) is the measurement yields $a_n$ then the quantum system is left in the eigenstate $|A=a_n\rangle$.

In the "weak" case (large $\Delta p$, small $\Delta$), this will approximate a single, broad Gaussian peaked at the mean value of $\hat{A}$, which is $\langle \hat{A} \rangle = \sum_n |\alpha_n|^2 a_n$. Of course, any single "weak measurement" gives almost no information, since $\Delta p \gg \langle \hat{A} \rangle$. However, by repeating the experiment many times one can map out the whole distribution, and so determine the centroid $\langle \hat{A} \rangle$ to any desired accuracy.[4]

In the opposite limit, where Δp is much bigger than all a;, the final probability distribution will be again close to a Gaussian with the spread Δp

# Mesure faible (3)

Post-Sélection : mesure forte autre grandeur.
On choisit l'état final If> de cette mesure

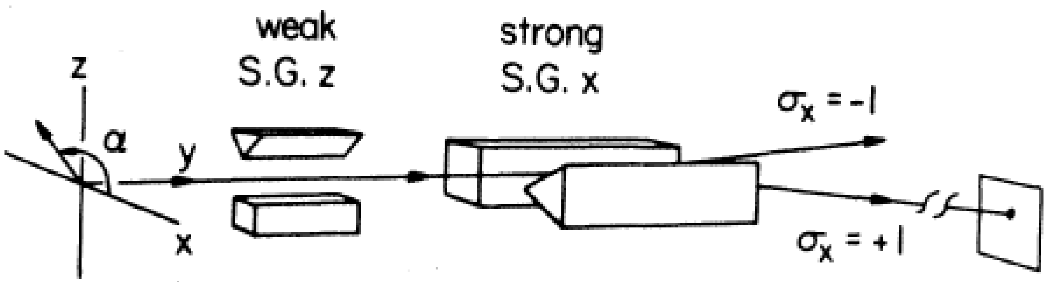
AAV « démontre que $$|\Phi_f\rangle = \sum_n \alpha_n \alpha_n'^* \int dp \, \exp[-\Delta^2(p-a_n)^2]|p\rangle$$

$$A_w \equiv \langle \Psi_f | \hat{A} | \Psi_{\text{in}} \rangle / \langle \Psi_f | \Psi_{\text{in}} \rangle$$

« Aiguille » = gaussienne largeur $\Delta$, centrée sur Aw

# Mesure faible en polarisation sur spin 1/2

Quantum system : spin
Aiguille : fonction onde position




Thus, the initial spin state is the $+1$ eigenstate of $\cos\alpha)\sigma_x + (\sin\alpha)\sigma_z$,

$$|\Psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos\frac{\alpha}{2} + \sin\frac{\alpha}{2} \\ \cos\frac{\alpha}{2} - \sin\frac{\alpha}{2} \end{bmatrix}, \tag{23}$$

and the final state is the $+1$ eigenstate of $\hat{\sigma}_x$:

$$|\Psi_f\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{24}$$

The initial spatial wave function is

$$\phi_{\text{in}}(q) \equiv \langle q|\Phi_{\text{in}}\rangle = \exp\left[-\frac{z^2}{4\Delta^2}\right] f(x,y). \tag{27}$$

The precise $x,y$ dependence is unimportant and we shall ignore it henceforward.

The corresponding exact result is easily obtained from Eq. (11):

$$\tilde{\phi}_f(p) = \varphi(p_z;\epsilon,\Delta,\lambda), \tag{32}$$

where $\varphi$ is a function of $p$, with parameters $\epsilon$, $\Delta$, and $\lambda$, defined by

$$\varphi(p;\epsilon,\Delta,\lambda) \equiv \tfrac{1}{2}\{(1+\epsilon)\exp[-\Delta^2(p-\lambda)^2] - (1-\epsilon)\exp[-\Delta^2(p+\lambda)^2]\}. \tag{33}$$

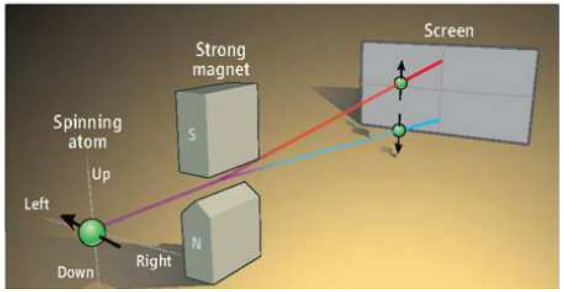
Note that $\lambda$ could be eliminated by the scaling relation

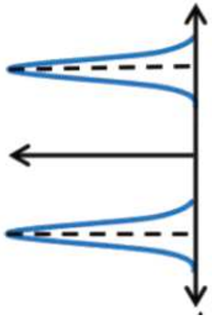
$$\varphi(p;\epsilon,\Delta,\lambda) = \varphi(p/\lambda;\epsilon,\lambda\Delta,1). \tag{34}$$

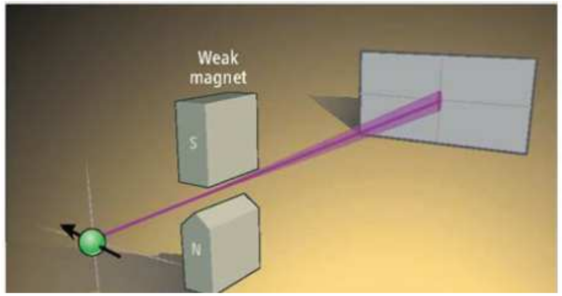
To understand this better let us specialize to the case where $\alpha = \pi - 2\epsilon$ with $\epsilon \ll 1$. AAV's result then reduces to

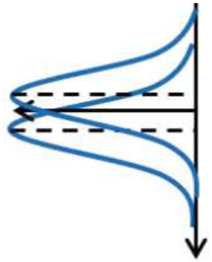
$$\tilde{\phi}_f(p) \simeq \epsilon \exp[-\Delta^2(p_z - \lambda/\epsilon)^2], \tag{30}$$

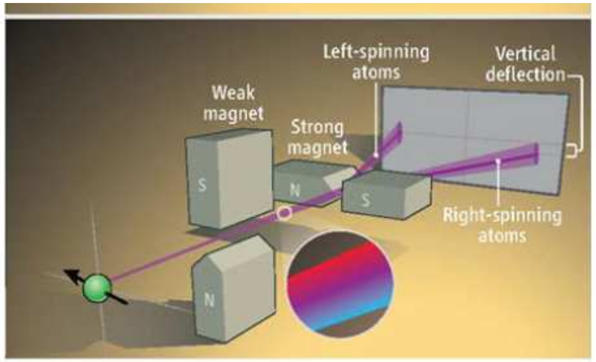
# Mesure faible en Stern et Gerlach

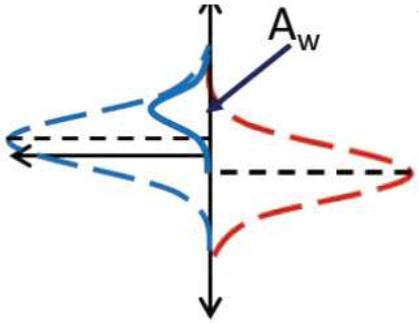


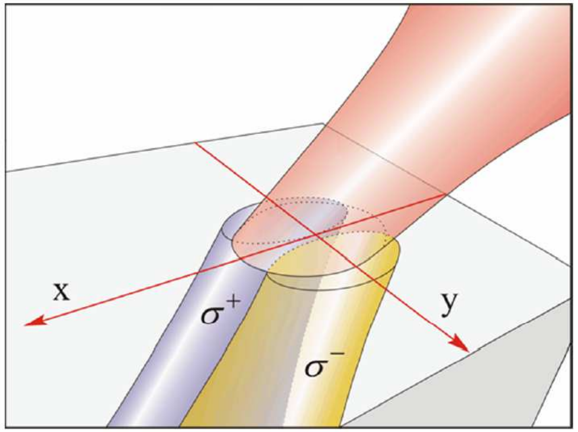


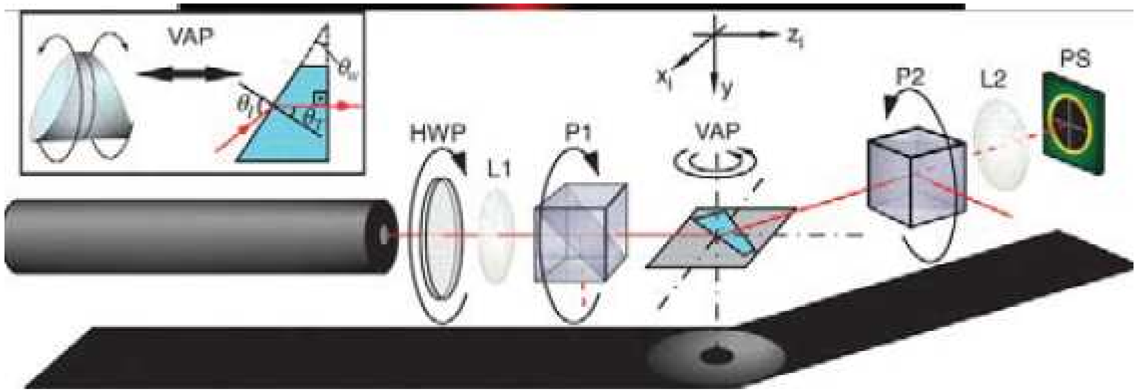


# Mesure faible et « shift » d'Imbert




**Observation of the Spin Hall Effect of Light via Weak Measurements**
Onur Hosten, *et al.*
*Science* **319**, 787 (2008);
DOI: 10.1126/science.1152697




A étudier plus tard à Epiphymath avec JMV

# Mesure faible en polarisation

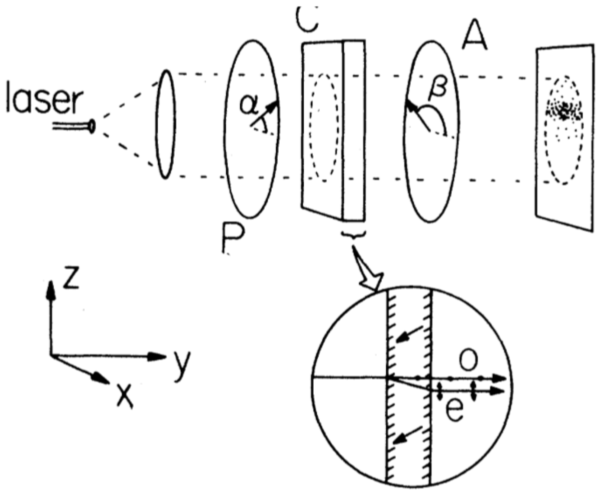



FIG. 5. An optical analog of the AAV experiment. A broad, coherent beam passes through a polarizer ($P$) and analyzer ($A$). Between them is a birefringent crystal ($C$) which produces a small lateral displacement between $x$ ($o$ ray) and $z$ ($e$ ray) polarizations (see inset).

The initial beam is assumed to have a Gaussian profile, with a large spatial width, $1/(2\delta)$. After passing through the polarizer, its wave function is

$$\langle q|\Phi_{in}\rangle|\Psi_{in}\rangle=\exp(-\delta^2 z^2)\begin{bmatrix}\cos\alpha\\ \sin\alpha\end{bmatrix}, \tag{39}$$

nor. In passing through the birefringent material the $o$ ray ($x$ polarization) and $e$ ray ($z$ polarization) are displaced by different amounts $a_1$ and $a_2$, so the emerging beam will be

$$\begin{bmatrix}\cos\alpha\exp[-\delta^2(z-a_1)^2]\\ \sin\alpha\exp[-\delta^2(z-a_2)^2]\end{bmatrix}. \tag{40}$$

Finally, the analyzer projects out the $\beta$ component of polarization, producing the spatial wave function

$$\langle q|\Phi_f\rangle=\cos\beta\cos\alpha\exp[-\delta^2(z-a_1)^2]+\sin\beta\sin\alpha\exp[-\delta^2(z-a_2)^2]. \tag{41}$$

# Mesure faible en polarisation

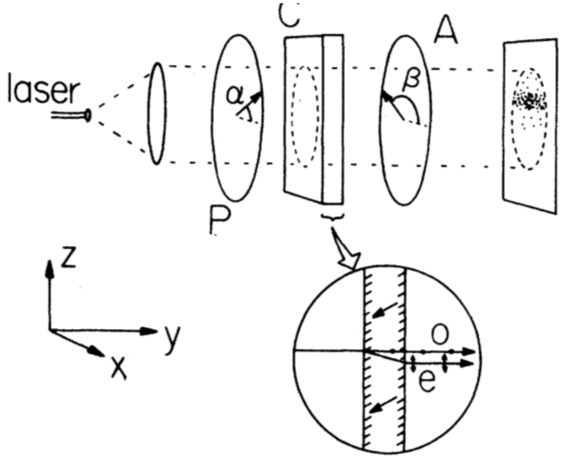



FIG. 5. An optical analog of the AAV experiment. A broad, coherent beam passes through a polarizer ($P$) and analyzer ($A$). Between them is a birefringent crystal ($C$) which produces a small lateral displacement between $x$ ($o$ ray) and $z$ ($e$ ray) polarizations (see inset).

Finally, the analyzer projects out the $\beta$ component of polarization, producing the spatial wave function

$$\langle q|\Phi_f\rangle = \cos\beta\cos\alpha\exp[-\delta^2(z-a_1)^2] + \sin\beta\sin\alpha\exp[-\delta^2(z-a_2)^2] \ . \qquad (41)$$

This can be rewritten as

$$\phi_f(q) \equiv \langle q|\Phi_f\rangle = \cos(\alpha+\beta)\varphi(z-\bar{a};\epsilon,\delta,\lambda) \qquad (42)$$

in terms of the $\varphi$ function introduced in Eq. (33). The parameters are given by

$$\bar{a} = \tfrac{1}{2}(a_1+a_2), \quad \lambda = \tfrac{1}{2}(a_1-a_2) \ , \qquad (43)$$

$$\epsilon = \epsilon_{(1)}(\alpha,\beta) \equiv \frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} \ .$$

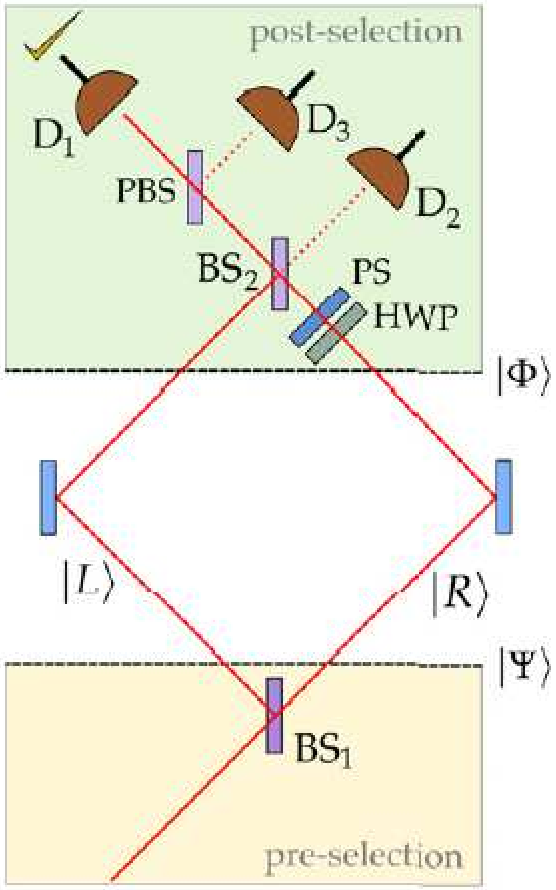
$$\varphi(p;\epsilon,\Delta,\lambda) \equiv \tfrac{1}{2}\{(1+\epsilon)\exp[-\Delta^2(p-\lambda)^2] - (1-\epsilon)\exp[-\Delta^2(p+\lambda)^2]\} \ .$$

A discuter en détail plus tard

## Quantum Cheshire Cats

**Yakir Aharonov Sandu Popescu, Daniel Rohrlich,Paul Skrzypczyk**

*New Journal of Physics* **15** (2013) 113015 (8pp)




We would like to post-select the state $|\Phi\rangle$,

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|L\rangle|H\rangle + |R\rangle|V\rangle).$$

send a horizontally polarized photon toward a 50:50 beam splitter,

Suppose that the photon is initially prepared in a state $|\Psi\rangle$,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(i|L\rangle + |R\rangle)|H\rangle,$$

# Quantum Cheshire Cats

**Yakir Aharonov Sandu Popescu, Daniel Rohrlich,Paul Skrzypczyk**

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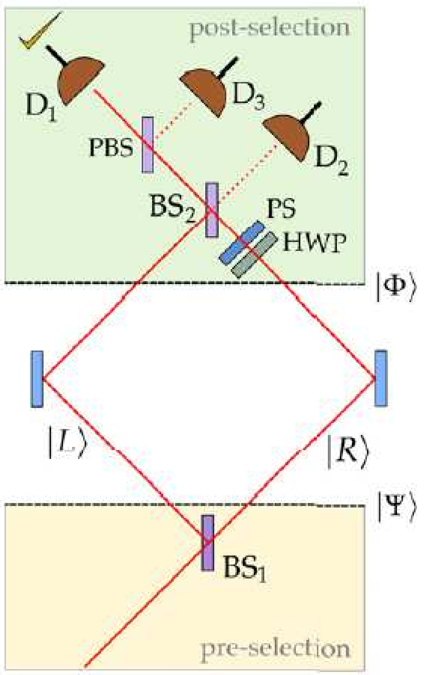
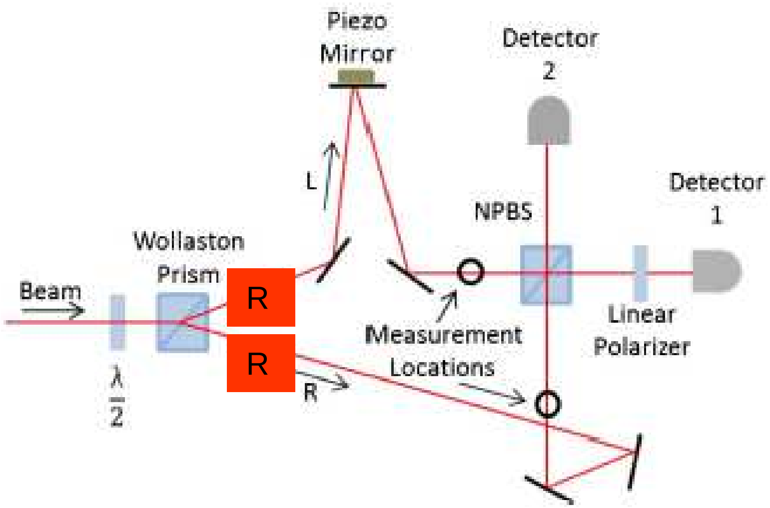



We would like to post-select the state $|\Phi\rangle$,

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|L\rangle|H\rangle + |R\rangle|V\rangle).$$

In other words, we would like to perform a final measurement that gives the answer 'yes' with certainty whenever the system is in the state $|\Phi\rangle$ and the answer 'no', again with certainty, whenever the state is orthogonal to $|\Phi\rangle$. We will then consider only those cases in which the answer 'yes' is obtained. Such a measurement can be experimentally realized in an optics setup, as depicted in figure 1. The measuring device comprises a half-wave plate (HWP), a phase shifter (PS), a beam splitter ($BS_2$), a polarizing beam splitter (PBS) and three photon detectors ($D_i$). The HWP is chosen such that $|H\rangle \leftrightarrow |V\rangle$. The PS is chosen to add a phase factor $i$ on the beam. $BS_2$ is chosen such that if a photon in the state $(|L\rangle + i|R\rangle)/\sqrt{2}$ impinges upon it, then it will certainly emerge from the left port (i.e. the detector $D_2$ will certainty *not* click). The PBS is chosen such that $|H\rangle$ is transmitted and $|V\rangle$ is reflected. Given these choices, if the state immediately before the HWP (i.e. the state of the photon entering the measuring device) is $|\Phi\rangle$, then $D_1$ will click with certainty. A photon in any state orthogonal to $|\Phi\rangle$ will end up either at detector $D_2$ or at $D_3$. We thus want to consider the experimental arrangement depicted

**Observation of a classical Cheshire cat in an optical interferometer**
David P. Atherton, Gambhir Ranjit, Andrew A. Geraci, and Jonathan D. Weinstein
March 15, 2015 / Vol. 40, No. 6 / OPTICS LETTERS 879




R Rotation

**Measurement location** = absorption

Indice 1 : voie 1
Indice 2 voie 2
Rotations (Teta1 , Teta2)
Absorption (a1,a2)
Phi1,Phi2 déphasages de propagation ou de transmission

```
Element[{Phi1, Phi2 , a1, a2, Teta1, Teta2}, Reals];
ex = {1, 0}; ey = {0, 1};
R[Teta_] = {{Cos[Teta], Sin[Teta]}, {-Sin[Teta], Cos[Teta]}};
Psi1 = a1 * Exp[I * Phi1] * R[Teta1].ex;
Psi2 = a2 * Exp[I * Phi2] * R[Teta2].ey;
Psi = Psi1 + Psi2;
Psif = ex;
r = ex.Psi ;
Print["Amplitude Photon = ", r]
```

Voie 1 polarisée x
Voie 2 polarisée y
Avant rotation

Postsélection selon x

$$\text{Amplitude} = a1e^{iPhi1}\text{Cos[Teta1]} + a2e^{iPhi2}\text{Sin[Teta2]}$$

$$\text{Intensité} = a1^2\,\text{Cos[Teta1]}^2 + 2\,a1\,a2\,\text{Cos[Phi1 - Phi2]}\,\text{Cos[Teta1]}\,\text{Sin[Teta2]} + a2^2\,\text{Sin[Teta2]}^2$$

# Discussion Chat Optique

$$Intensité = a1^2 \,Cos[Teta1]^2 + 2\, a1\, a2\, Cos[Phi1 - Phi2]\, Cos[Teta1]\, Sin[Teta2] + a2^2\, Sin[Teta2]^2$$

1) Teta1=Teta2=0 : absorption sur voie 1 seule (Polarisation =x pour Postsélection)
2) a1=a2=1; Teta1 et Teta2 petits; I= 1+2cos(Phi)Teta2

interférence dépend de l'angle voie 2 seule (Polar y)

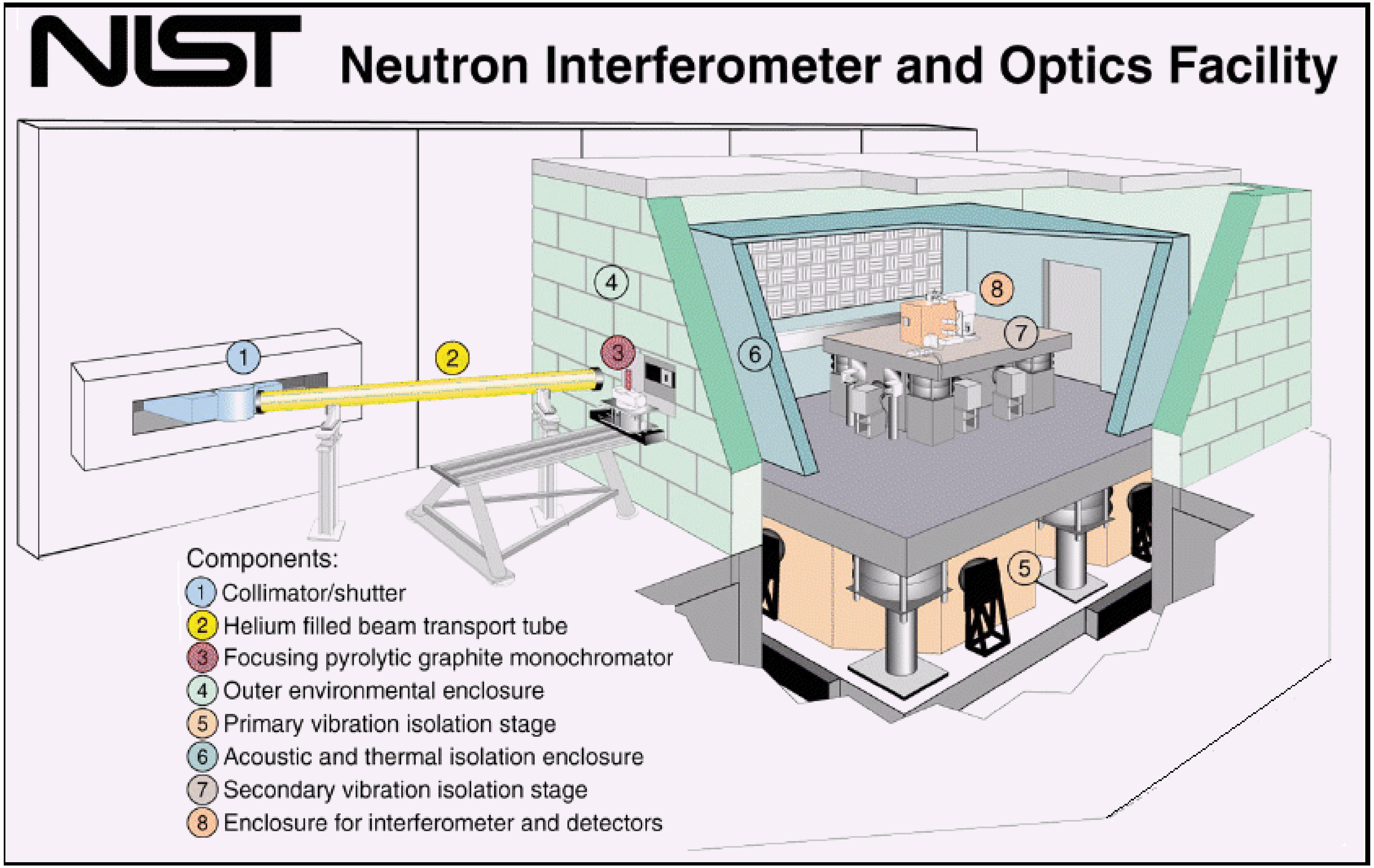
3) Absorptions et rotations petites

$Int \cong a1^2 + 2\, a1\, a2\, Cos[Phi1-Phi2]\, Teta2$
absorption 1-a1
absorption 1-a2

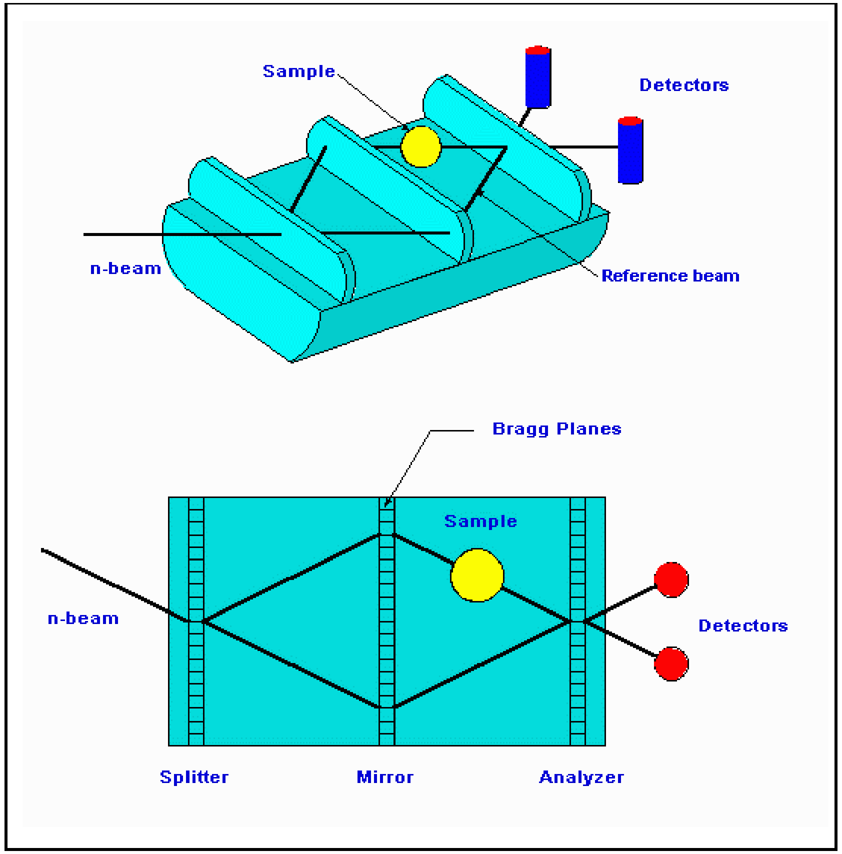
$Int \cong a1^2 + 2\, a1\, a2\, Cos[Phi1-Phi2]\, Teta2+$

Intensité sur voie 1
Rotation sur voie 2

# Interféromètre à Neutrons Polarisés



# National Institute of Standards and Technologies

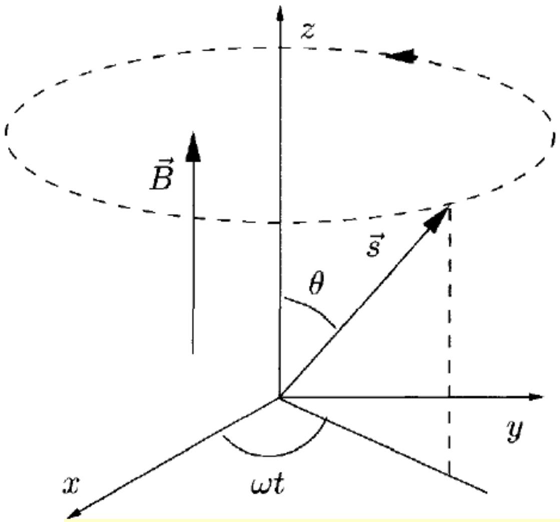



The Neutron Interferometer Facility in the Cold Neutron Guide Hall became operational in April 1994. It became available as a National User Facility in September 1996. Phase contrast of up to 88 percent and phase stability of better than five milliradians per day were observed. These performance indications are primarily the result of the advanced vibration isolation and environmental control systems. The interferometer operates inside a double walled enclosure, with the inner room built on a 40,000 kg slab which floats on pneumatic pads above an isolated foundation.

# Interféromètres

Photograph of the symmetric (left) and skew-symmetric (right) interferometers used in [22]. The blades of the symmetric interferometer are 3.077 mm thick and 50.404 mm apart. The dimensions of the skew-symmetric interferometer are 16.172 and 49.449 mm. The blade thickness is 2.621 mm. The photograph is reprinted with permission from Littrell K C et al 1997 Phys. Rev. A 56 1767 [22]. Copyright 1997 by the American Physical Society.

# Précession de Larmor




**Moment cinétique :** $\vec{\sigma}$

**Couple :** $\vec{\Gamma} = \vec{\mu} \wedge \vec{B}$

**Moment Magnétique :** $\vec{\mu}$

**Champ Magnétique constant :** $\vec{B}$

$\vec{\mu} = \gamma \vec{\sigma}$

**Relation Fondamentale de la dynamique**

$$\frac{\mathbf{d}\vec{\sigma}}{\mathbf{dt}} = \vec{\Gamma} = \vec{\mu} \wedge \vec{B} \Rightarrow \boxed{\frac{\mathbf{d}\vec{\sigma}}{\mathbf{dt}} = \frac{1}{\gamma} \vec{\sigma} \wedge \vec{B}}$$

$$\vec{\sigma}\,\mathbf{.d}\vec{\sigma} = \mathbf{0} \Rightarrow \vec{\sigma}.\vec{\sigma} = \mathbf{Cste} \Rightarrow \sigma = Cst \text{ en module}$$

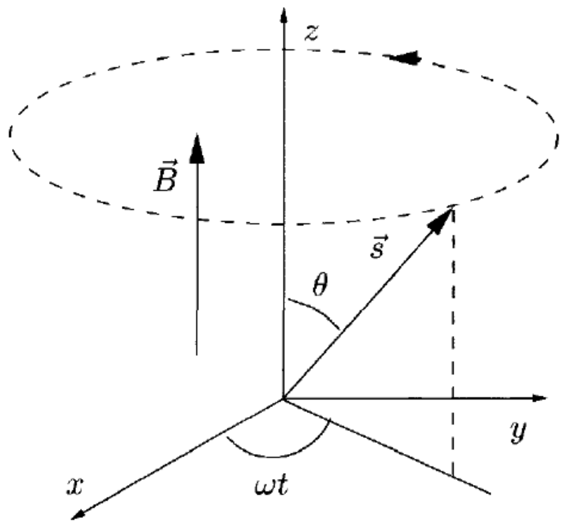
$$\vec{B}\,\mathbf{.d}\vec{\sigma} = \mathbf{0} \Rightarrow \vec{B}.\vec{\sigma} = \mathbf{Cste} = \mathbf{B}\sigma\mathbf{cos}(\theta) \Rightarrow \theta = Cst$$

Rotation en mécanique
D'un vecteur de module constant

$$\boxed{\frac{\mathbf{d}\vec{\sigma}}{\mathbf{dt}} = \vec{\omega_L} \wedge \vec{\sigma}}$$

$$\boxed{\vec{\omega_L} = -\gamma \vec{B}}$$

# Rapport gyromagnétique γ




$$\vec{\mu} = \gamma\vec{\sigma} = g\frac{q}{2m}\vec{\sigma}$$

Valeurs NIST du facteur de Landé[1].

| particule | $g_{particule}$ | Δ$g$ |
|---|---|---|
| électron | −2,002 319 304 3718 | 0,000 000 000 0075 |
| neutron | −3,826 085 46 | 0,000 000 90 |
| proton | +5,585 694 701 | 0,000 000 056 |
| muon | −2,002 331 8396 | 0,000 000 0012 |

Traversée d'un champ magnétique par un spin = rotation du spin

$$\alpha = -\frac{2\mu}{\hbar}\int_0^\tau B\,dt = -\frac{2\mu}{\hbar}B\frac{L}{v},$$

Rotation proportionnelle à
B
Distance parcourue dans B sur vitesse

# Spin flip :

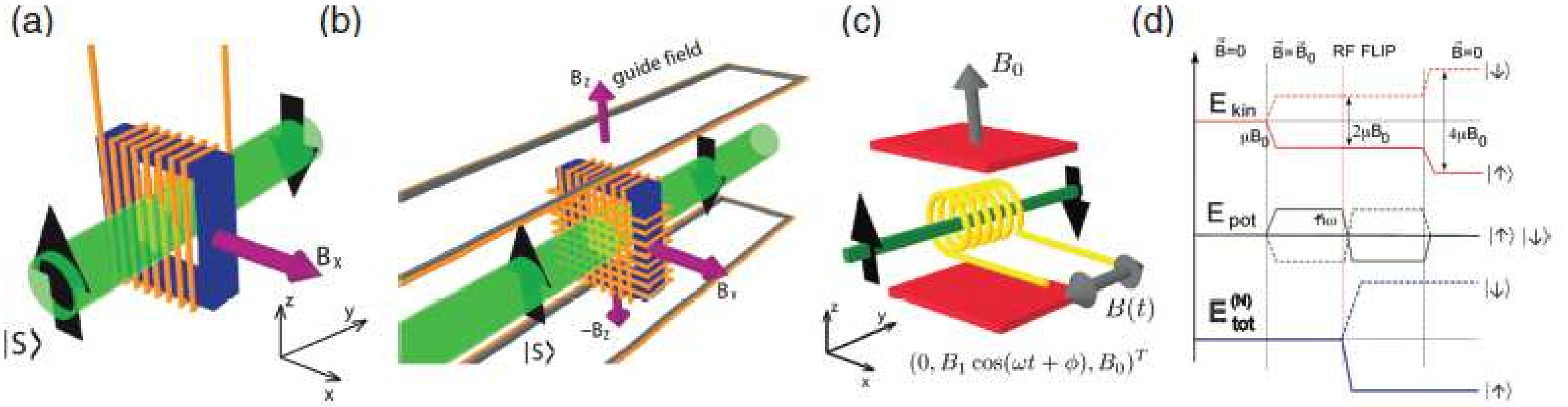
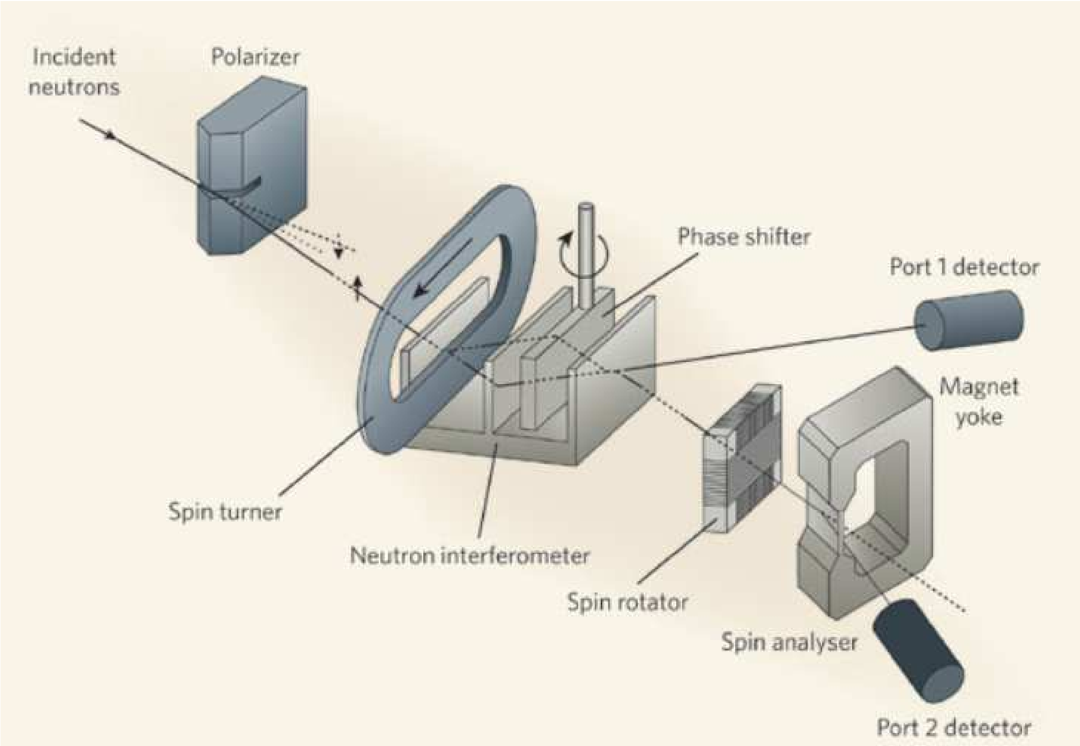



**Fig. 5.** (a) DC spin-flipper functional principle. (b) In practice, a second coil with its field pointing in the $-z$ direction (perpendicular to the original coil) is necessary to compensate the guide field. (c) Combination of static and oscillating magnetic fields. (d) Energy scheme for the RF-flip process.

Electroaimant : rotation réglable
Absorption onde radio = inversion du spin (spin haut devient bas)

# Interféromètre avec Rotateur




Hasegawa and colleagues' latest experiments2 on quantum reality reuse an earlier experimental set-up of theirs8. Neutrons are spin-polarized before they enter the interferometer, where the neutron beam is split into two possible paths before recombining and exiting through one of two ports. An entanglement between path and spin is established by changing the spin of the neutrons on one of the paths. The phase shifter determines the type of port measurement by exploiting neutron interference at the point where the two beams recombine. A spin rotator after the interferometer selects the direction in which spin is measured for one of the beams. The correlations between spin and exit port measurements can be determined from the numbers of neutrons detected per unit time for each setting. (Figure after ref. 8.)

# Interféromètre à neutrons du Chat

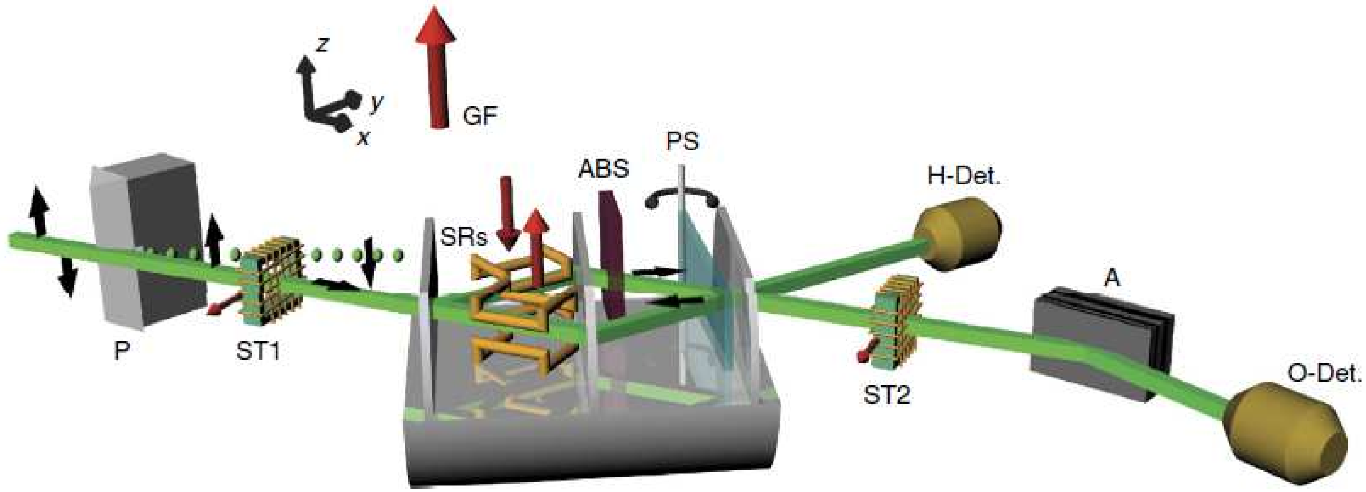



**Figure 2 | Illustration of the experimental setup.** The neutron beam is polarized by passing through magnetic birefringent prisms (P). To prevent depolarization, a magnetic guide field (GF) is applied around the whole setup. A spin turner (ST1) rotates the neutron spin by $\pi/2$. Preselection of the system's wavefunction $|\psi_i\rangle$ is completed by two spin rotators (SRs) inside the neutron interferometer. These SRs are also used to perform the weak measurement of $\langle\hat{\sigma}_z\hat{\Pi}_I\rangle_w$ and $\langle\hat{\sigma}_z\hat{\Pi}_{II}\rangle_w$. The absorbers (ABS) are inserted in the beam paths when $\langle\hat{\Pi}_I\rangle_w$ and $\langle\hat{\Pi}_{II}\rangle_w$ are determined. The phase shifter (PS) makes it possible to tune the relative phase $\chi$ between the beams in path $I$ and path $II$. The two outgoing beams of the interferometer are monitored by the H and O detector in reflected and forward directions, respectively. Only the neutrons reaching the O detector are affected by postselection using a spin turner (ST2) and a spin analyzer (A).

# Publi chat avec neutrons : Absorbeur sans Rotation

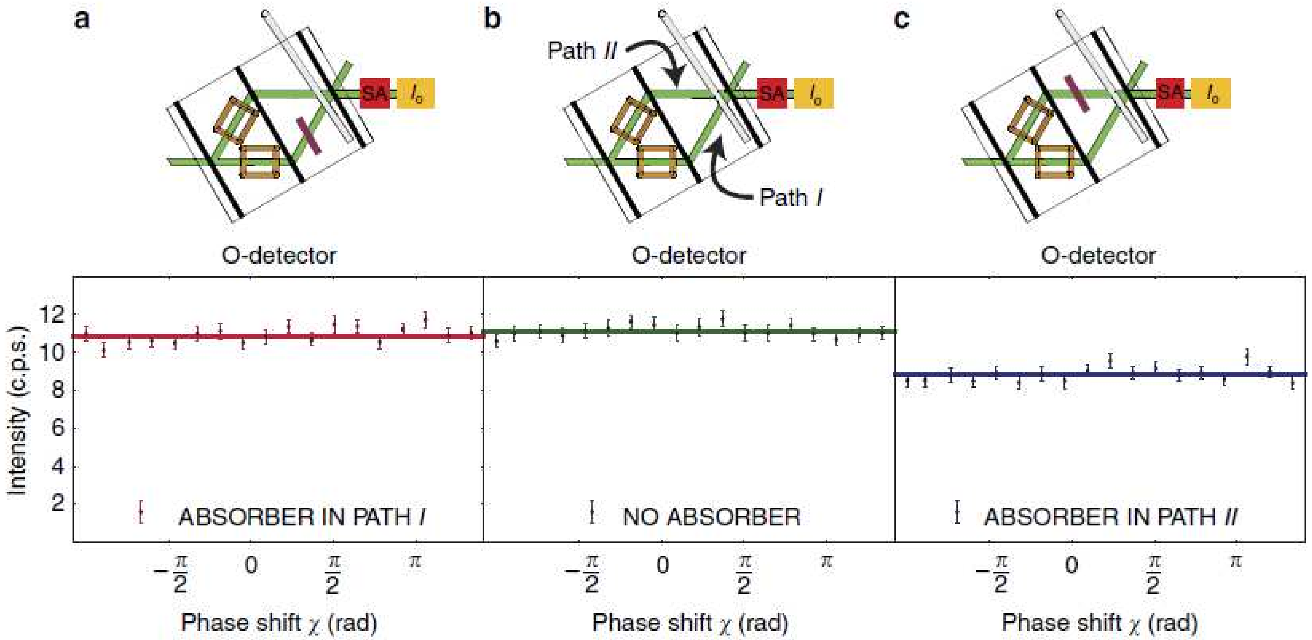



**Figure 3 | Measurement of $\langle\hat{\Pi}_I\rangle_w$ and $\langle\hat{\Pi}_{II}\rangle_w$ using an absorber with transmissivity $T = 0.79(1)$.** The intensity is plotted as a function of the relative phase $\chi$. The solid lines represent least-square fits to the data and the error bars represent one s.d. (**a**) An absorber in path *I*; no significant loss in intensity is recorded. (**b**) A reference measurement without any absorber. (**c**) An absorber in path *II*: the intensity decreases. These results suggest that for the successfully postselected ensemble, the neutrons go through path *II*.

# Publi chat avec neutron : Rotations

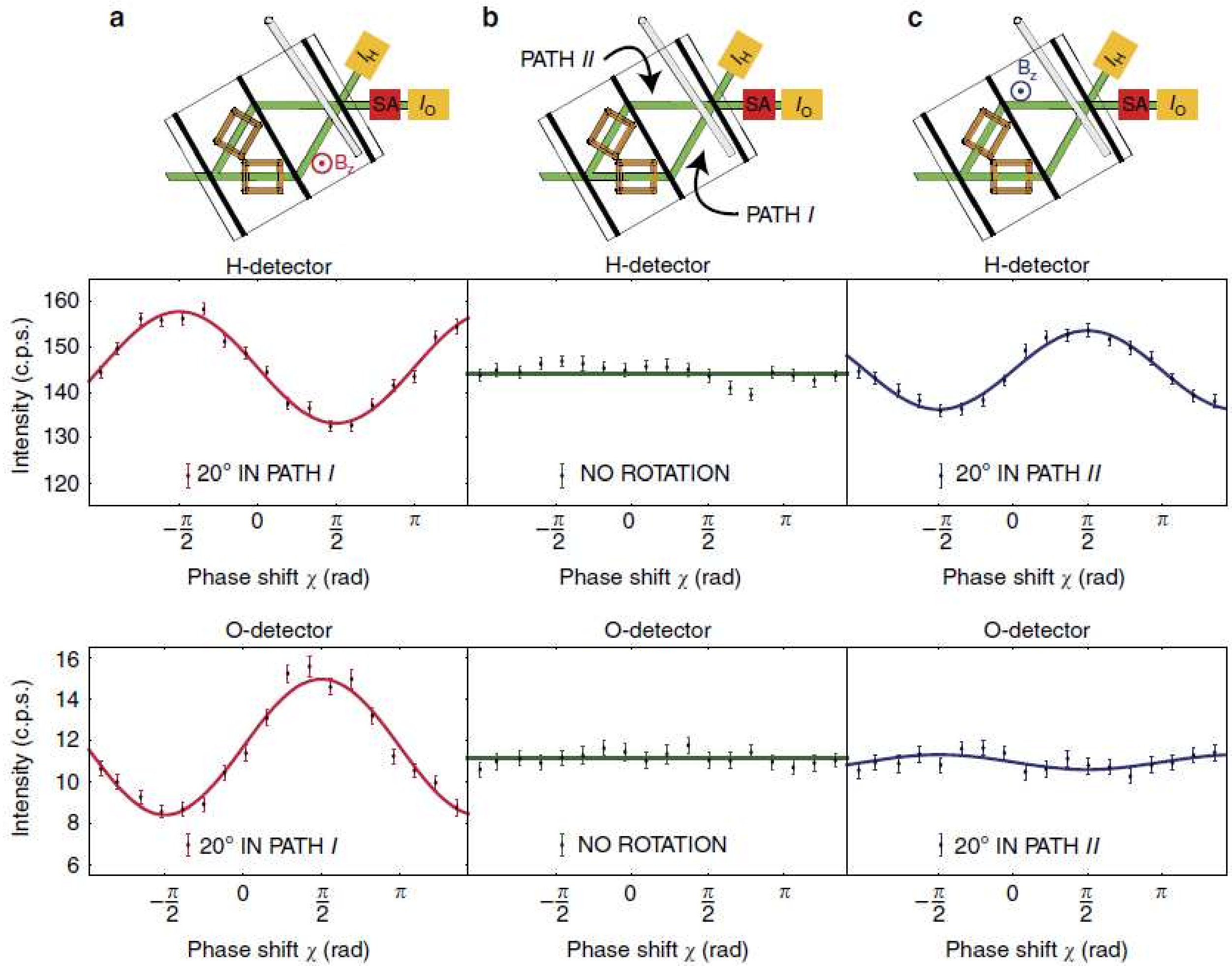
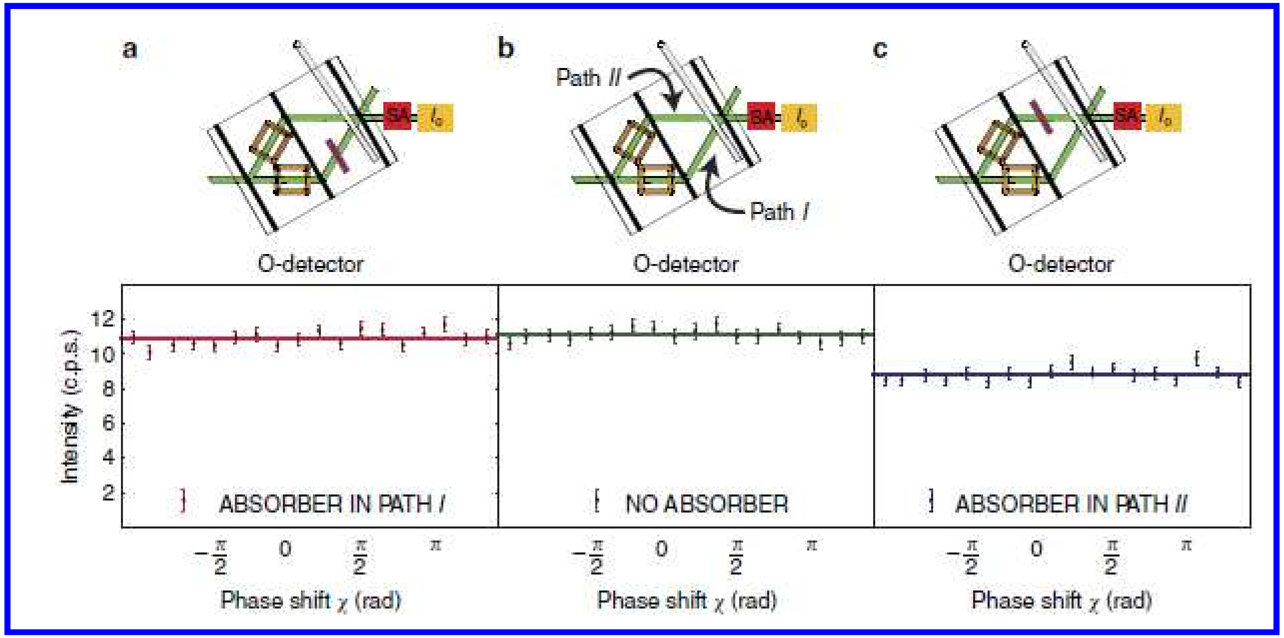
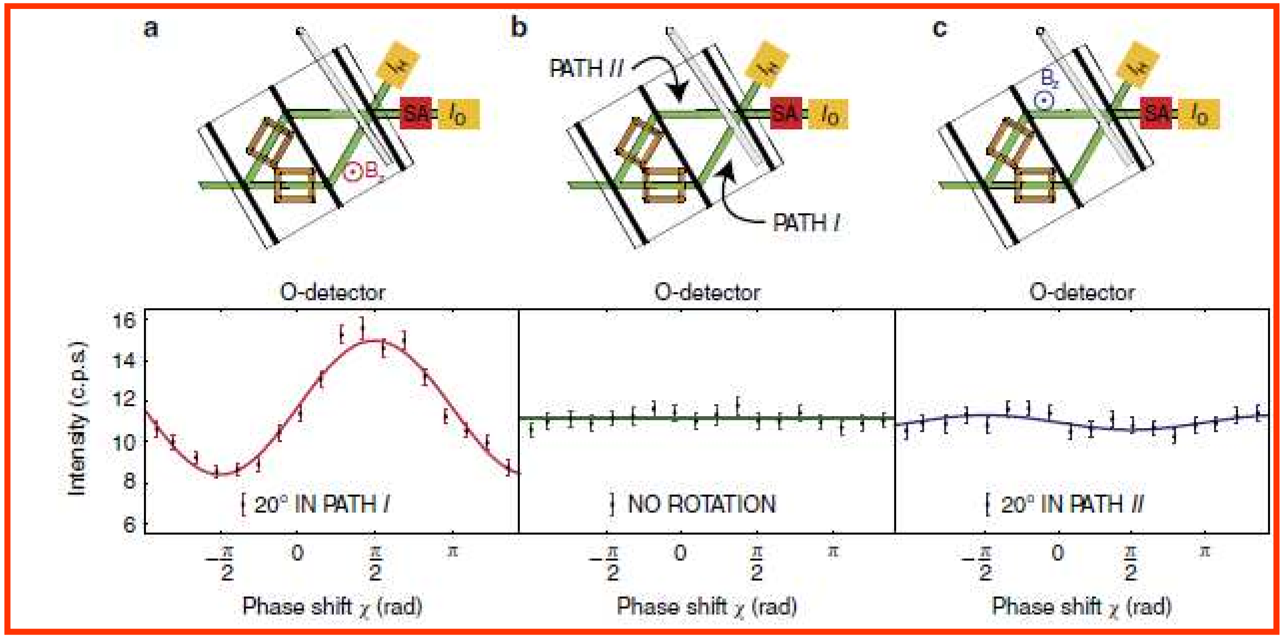



**Figure 4 | Measurement of $\langle \hat{\sigma}_z \hat{\Pi}_I \rangle_w$ and $\langle \hat{\sigma}_z \hat{\Pi}_{II} \rangle_w$ applying small additional magnetic fields.** The intensity of the O beam (with the spin analysis) and the H beam (without the spin analysis) is plotted as a function of the relative phase $\chi$. The solid lines represent least-square fits to the data and the error bars represent one s.d. (**a**) A magnetic field in path *I*; interference fringes appear both at the postselected O detector and the H detector. (**b**) A reference measurement without any additional magnetic fields. Since the spin states inside the interferometer are orthogonal, interference fringes appear neither in the O, nor the H detector. (**c**) A magnetic field in path *II*; interference fringes with minimal contrast can be seen at the spin postselected O detector, whereas a clear sinusoidal intensity modulation is visible at the H detector without spin analysis. The measurements suggest that for the successfully postselected ensemble (only the O detector) the neutrons' spin component travels along path *I*.



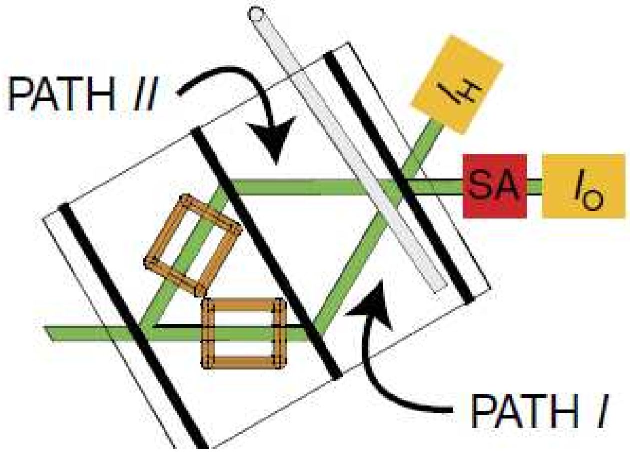

**Pour détection Polarisée Voie O**

**Absorption : Path II**




**Rotation : Path I**

# Calcul du chat du Cheshire avec spin 1/2




Rotation sur les deux voies
Atténuation sur les deux voies
Path1 : spin haut
Path 2 : spin bas
Postsélection : spin bas

```
Sx = PauliMatrix[1]; Sy = PauliMatrix[2]; Sz = PauliMatrix[3];
Psi[z, 1] = {1, 0}; Psi[z, -1] = {0, -1};
Psi[x, 1] = {1, 1} / Sqrt[2]; Psi[x, -1] = {-1, 1} / Sqrt[2];
Psi[y, 1] = {-I, 1} / Sqrt[2]; Psi[y, -1] = {I, 1} / Sqrt[2];
R[Teta_] = MatrixExp[I * Teta * Sz / 2];
Psi1 = a1 * Exp[I * Phi1] * R[Teta1].Psi[x, 1] ;
Psi2 = a2 * Exp[I * Phi2] * R[Teta2].Psi[x, -1];
Psit = Psi1 + Psi2;
Psif = Psi[x, -1];
```

$$\text{Amplitude} = a2\, e^{i\,\text{Phi2}} \text{Cos}\left[\frac{\text{Teta2}}{2}\right] + a1\,(-i\,\text{Cos}[\text{Phi1}] + \text{Sin}[\text{Phi1}])\,\text{Sin}\left[\frac{\text{Teta1}}{2}\right]$$

$$\text{Intensité} = \frac{1}{2} a2^2 (1 + \text{Cos}[\text{Teta2}]) + 2\, a1\, a2\, \text{Cos}\left[\frac{\text{Teta2}}{2}\right] \text{Sin}[\text{Phi1} - \text{Phi2}]\, \text{Sin}\left[\frac{\text{Teta1}}{2}\right] + a1^2\, \text{Sin}\left[\frac{\text{Teta1}}{2}\right]^2$$

Path 2 : absorption

Path 1 : Rotation

# Programme Mathématica chat neutron

## Neutron – Electron

```
Element[{Phi1, Phi2 , a1, a2, Teta1, Teta2}, Reals];
Sx = PauliMatrix[1]; Sy = PauliMatrix[2]; Sz = PauliMatrix[3];
Psi[z, 1] = {1, 0}; Psi[z, -1] = {0, -1};
Psi[x, 1] = {1, 1} / Sqrt[2]; Psi[x, -1] = {-1, 1} / Sqrt[2];
Psi[y, 1] = {-I, 1} / Sqrt[2]; Psi[y, -1] = {I, 1} / Sqrt[2];
R[Teta_] = MatrixExp[I * Teta * Sz / 2];
Psi1 = a1 * Exp[I * Phi1] * R[Teta1].Psi[x, 1] ;
Psi2 = a2 * Exp[I * Phi2] * R[Teta2].Psi[x, -1];
Psit = Psi1 + Psi2;
Psif = Psi[x, -1];
r = FullSimplify[ComplexExpand[Conjugate[Psif].Psit], Element[{Phi1, Phi2 , a1, a2, Teta1, Teta2}, Reals]]; Print["Amplitude = ", r];
rc = FullSimplify[ComplexExpand[Conjugate[r]], Element[{Phi1, Phi2 , a1, a2, Teta1, Teta2}, Reals]] ;
Ir = ExpToTrig[Expand[r * rc]];
Ir1 = Collect[Ir, {a1, a2}];
Ir2 = FullSimplify[Ir1, Element[{Phi1, Phi2 , a1, a2, Teta1, Teta2}, Reals]];


Print["Intensité = ", Ir2]
```

Opérateur rotation de spin 1/2 (R[Teta_])

Voie 1, spin haut avant rotation (Psi1)

Voie 2, spin bas avant rotation (Psi2)

Postsélection (Psif)

Amplitude = $a2\, e^{i\, Phi2} \mathrm{Cos}\left[\frac{Teta2}{2}\right] + a1\,(-i\,\mathrm{Cos}[Phi1] + \mathrm{Sin}[Phi1])\,\mathrm{Sin}\left[\frac{Teta1}{2}\right]$

Intensité = $\frac{1}{2}\, a2^2\,(1 + \mathrm{Cos}[Teta2]) + 2\, a1\, a2\, \mathrm{Cos}\left[\frac{Teta2}{2}\right] \mathrm{Sin}[Phi1 - Phi2]\, \mathrm{Sin}\left[\frac{Teta1}{2}\right] + a1^2\, \mathrm{Sin}\left[\frac{Teta1}{2}\right]^2$

# Discussion : chat avec neutron

$$\text{Intensité} = \frac{1}{2}\, a2^2 \left(1 + \text{Cos}[\text{Teta2}]\right) + 2\, a1\, a2\, \text{Cos}\left[\frac{\text{Teta2}}{2}\right] \text{Sin}[\text{Phi1} - \text{Phi2}]\, \text{Sin}\left[\frac{\text{Teta1}}{2}\right] + a1^2\, \text{Sin}\left[\frac{\text{Teta1}}{2}\right]^2$$

Voie 2 en $\cos^2$(Teta2/2)
absorption

Voie 1 en $\sin^2$(Teta1/2)
Rotation

**Formule identique à l'optique (sauf Teta-→Teta/2 à cause du spin)**
**Discussion semblable**
« Tout se passe comme si le spin passe par la voie 1 et la matière par la voie 2 »
C'est vrai, mais effet d'interférence avec choix judicieux de polarisation.
L'expression : tout se passe comme si est judicieusement choisie

# Plus intéressant que l'expérience du Chat du Cheshire

Dans quel espace est le Psi quantique ?

# Opérateurs de Spin 1/2

Opérateurs de spin 1/2 $\hat{S}_{x,y,z} \equiv \frac{\hbar}{2}\sigma_{x,y,z}$ Matrices de Pauli

$$\left[\hat{S}_x, \hat{S}_y\right] = \hat{S}_x\hat{S}_y - \hat{S}_y\hat{S}_x = i\hbar\hat{S}_z$$

$$\left[\hat{S}_y, \hat{S}_z\right] = i\hbar\hat{S}_x,$$

$$\left[\hat{S}_z, \hat{S}_x\right] = i\hbar\hat{S}_y$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Vecteurs Propres

$$\text{Psi}[z,1] = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{Psi}[z,-1] = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\text{Psi}[x,1] = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{Psi}[x,-1] = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\text{Psi}[y,1] = \begin{pmatrix} -\frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{Psi}[y,-1] = \begin{pmatrix} \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

# Algèbre des Matrices de Pauli

Les matrices de Pauli obéissent aux relations de commutation et d'anticommutation suivantes :

$$[\sigma_i, \sigma_j] = 2i\,\epsilon_{ijk}\,\sigma_k$$

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij} \cdot I$$

- $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- $\sigma_1\sigma_2 = i\sigma_3$
- $\sigma_3\sigma_1 = i\sigma_2$
- $\sigma_2\sigma_3 = i\sigma_1$
- $\sigma_i\sigma_j = -\sigma_j\sigma_i$ pour $i \neq j$

Formules faciles à vérifier

Formules à démontrer

$$\exp\left(-\mathrm{i}\frac{\theta}{2}\,\vec{\sigma}\cdot\hat{n}\right) = I\,\cos\frac{\theta}{2} - \mathrm{i}(\vec{\sigma}\cdot\hat{n})\,\sin\frac{\theta}{2}$$

# Dans quel espace « est » la fonction d'onde

Spin ½ (Matrices de Pauli) engendre groupe SU(2)
Espace des matrices Unitaires de déterminant 1.
Ces matrices travaillent dans un espace de Hilbert de dimension 2
**Les fonctions d'onde de spin sont dans cet espace de Hilbert**

$$\left\| \Psi \right\rangle \in \mathbf{HSU(2)}$$

Chaque opérateur de spin Sx, Sy,Sz agit dans SU(2)

$$\vec{S} = (S_x, S_y, S_z) \ \textbf{est un vecteur de l'espace ordinaire} \ (\mathbf{R}^3)$$

$$\textbf{et un opérateur vectoriel agissant dans HSU(2)}$$

# Opérateur vectoriel et Rotation

**Espace à 3 dimensions : Rotation $\mathbf{R}(\vec{\mathbf{n}},\theta)$; Ex : Rotation** $\mathbf{R}(\mathbf{z},\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$

**Grandeur vectorielle** : $\vec{\mathbf{V}}$ **Transformation par rotation** $\vec{\mathbf{V}} \xrightarrow{R} \vec{\mathbf{V}}' = R\vec{\mathbf{V}}$

**Représentation de la rotation dans SU(2)** : $\mathbf{R}(\vec{\mathbf{n}},\theta) \rightarrow \mathbf{U}(\vec{\mathbf{n}},\theta)$

**Conservation des relations de groupe** $\begin{cases} \mathbf{R}_1\mathbf{R}_2 \rightarrow \mathbf{U}_1\mathbf{U}_2 \\ 1 \rightarrow 1 \\ (\mathbf{R})^{-1} \rightarrow (\mathbf{U})^{-1} \end{cases}$

En plus U est unitaire : U∈ $SU(2)$
$UU^{\dagger} = U^{\dagger}U = 1$

Opérateur vectoriel : Transformation par rotation

$$\vec{\mathbf{V}} \xrightarrow{R} \vec{\mathbf{V}}' = \boxed{R(\vec{n},\theta)\vec{\mathbf{V}}} = \boxed{U^{+}(\vec{n},\theta)\vec{V}U(\vec{n},\theta)}$$

Dans R3 Dans SU(2)

# Rotation dans SU(2)

$$U(\vec{n},\theta) = \exp^{-\frac{i\theta}{\hbar}\vec{S}.\vec{n}} = \exp^{-\frac{i\theta}{2}\vec{\sigma}.\vec{n}} = \cos(\frac{\theta}{2}) - i\vec{\sigma}.\vec{n}\sin(\frac{\theta}{2})$$

**Transformation d'un opérateur de SU(2) :** $\mathbf{G} \rightarrow \mathbf{G'} = \mathbf{U^{+}GU}$

**Transformation d'un vecteur de HSU2 :** $|\Psi\rangle \rightarrow |\Psi'\rangle = U|\Psi\rangle$

**Propriétés curieuses et intéressantes**

$$U(\vec{n},2\pi) = -1$$

$$U(\vec{n},4\pi) = 1$$

**Rotation de** $2\pi$ **:** $|\Psi\rangle \xrightarrow{2\pi} |\Psi'\rangle = -|\Psi\rangle$

**Rotation de** $4\pi$ **:** $|\Psi\rangle \xrightarrow{4\pi} |\Psi'\rangle = |\Psi\rangle$

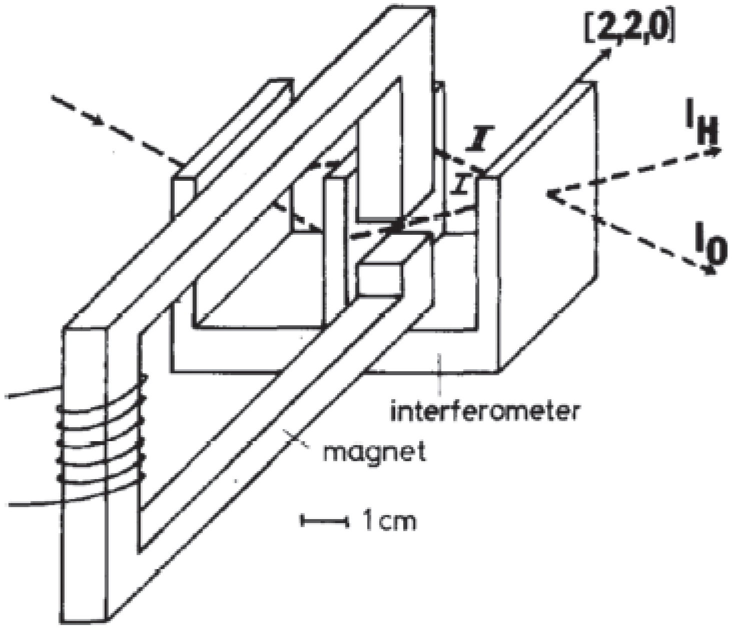
!!!!!!!!!!!

**Peut on mesurer ces propriétés directement ??**

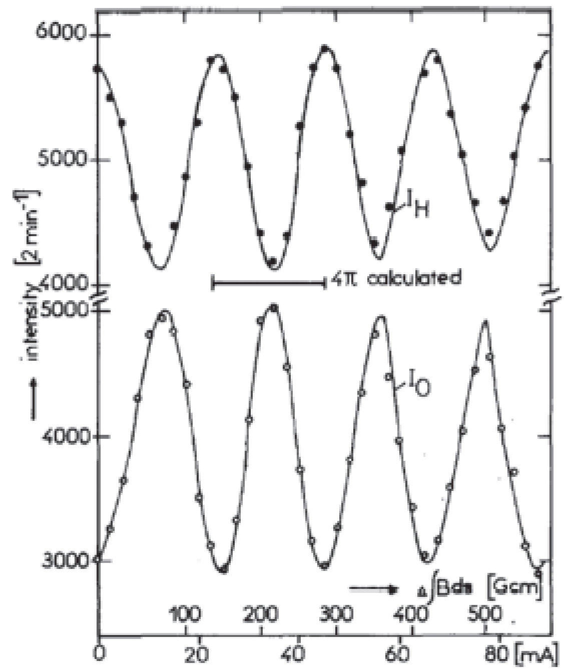
**Precise Determination of the 4 π-Periodicity Factor of a Spinor Wave Function**

H. Rauch and A. Wilting,W. Bauspiess,U. Bonse

Z. Physik B 29, 281-284 (1978)




$$\alpha = -\frac{2\mu}{\hbar}\int_0^{\tau} B\,dt = -\frac{2\mu}{\hbar}B\frac{L}{v},$$




Période = 716.8 +_3.8 deg,

# Addition spin up spin down

Qu'obtient-on en ajoutant spin up et spin down ?

Vecteurs Propres

$$\text{Psi}[z,1] = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{Psi}[z,-1] = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\text{Psi}[x,1] = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{Psi}[x,-1] = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\left|\Psi(z,1)\right\rangle + \left|\Psi(z,-1)\right\rangle = (1,1) = \sqrt{2}\left|\Psi(x,1)\right\rangle$$

**Peut-on observer directement cette propriété ??**

# Direct observation of fermion spin superposition by neutron interferometry

J. Summhammer, G. Badurek, H. Rauch, U. Kischko, A. Zeilinger

Phys. REV. A. 27 p2523 (1983)

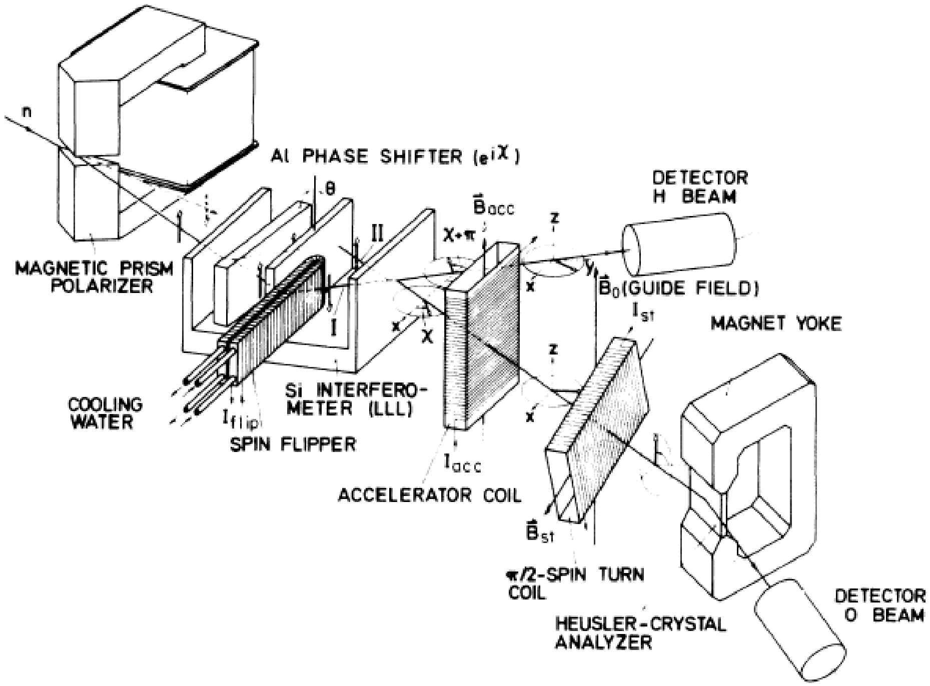



FIG. 2. Schematic of the experiment for spin superposition.

# Conclusion

- Chat de Cheshire = Simplement effet d'interférence
- Mais : mesure faible avec choix judicieux de polarisation

  ouvre des perspectives de mesures intéressantes

Mesures d'interférences avec neutons beaucoup plus intéress

rotation de 2Pi =multiplication par -1

Spin z up+Spin z down = Spin x haut

- Et bien d'autres expériences très intéressantes et fondamen
  moins connues que le chat, et effectuées depuis au moins 20

Question ouverte à débattre : « dans quelle espace « est » la fonction d'onde ?