Chap 6 :Magnetic Shape-memory Alloys Behavior

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Introduction.

Actuation by stress and temperature as classical SMA BUT ALSO BY MAGNETIC FIELD.
Response time around one millisecond for phase transformation in comparison with classical SMA: one second.
Mainly studied Ni-Mn-Ga and also Fe-Pd.
For actuation mainly martensite platelets rearrangement

Some models of the thermo-magneto-mechanical behavior of MSMAs

Two teams of specialists ; magnetism OR physics and strenghts of materials.
A distinction between models is to be founded in the CHOICE OF THE SCALE EXAMINED
O'Handley and Murray :two energies - mechanical -magnetism
Micromagnetism :changes of magnetic microstructure as a function of magnetic field
Likhachev and Ullakko introduce magnetic anisotropy

Crystallography of Ni-Mn-Ga

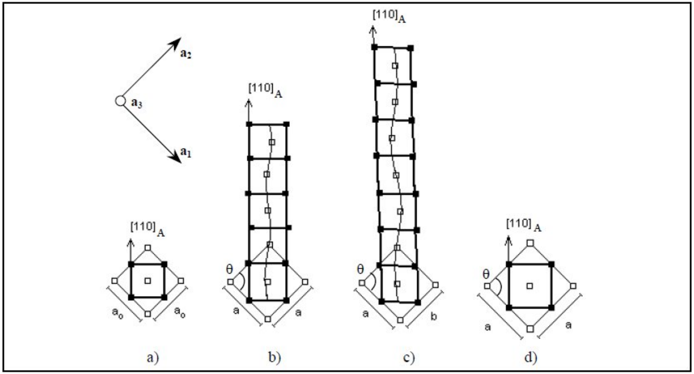


FIG.: Crystallographic structures of Ni-Mn-Ga : a) Austenite $L2_1$; b) Modulated quadratic martensite 5M) ;c)Modulated monoclinic martensite (7M) ; d)Non-modulated quadratic martensite (NMT) .

A 5M- type martensite may be present in the form of three variants

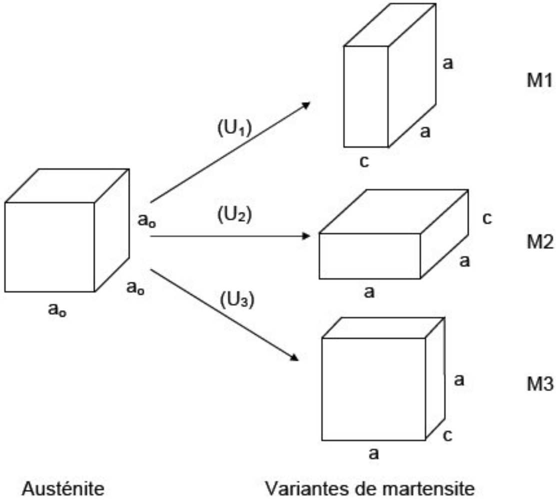


FIG.: The three martensite variants with a transformation from a cubic lattice into quadratic lattices.

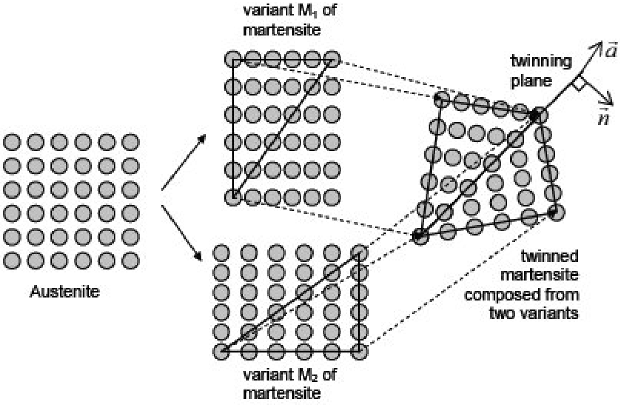


FIG.: A schematic two-dimension situation : one cubic (left) and two martensite variants M_1et M_2 side by side

Rearrangement and transformation

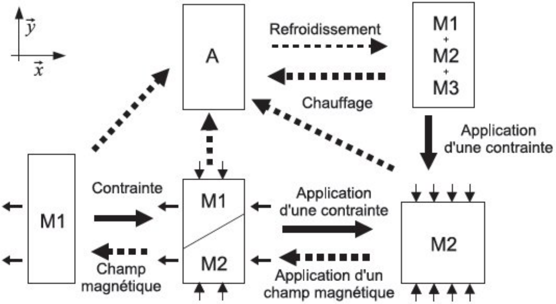


FIG.: Schematic visualization of a transformation and a martensitic rearrangement

Calculations of microstructures

Let $\mathbf{F}_{\mathrm{k}}$ be the gradient tensor of the transformation of austenite A into the variant k of martensite M

$$d\mathbf{x}(\mathrm{M}) = \mathbf{F}_{\mathrm{k}} d\mathbf{x}_0(\mathrm{A}) \tag{1}$$

and the Green - Lagrange tensor is defined by

$$\mathbf{E}_{\mathbf{k}}^{\mathrm{tr}} = \frac{1}{2}\left({}^{t}\mathbf{F}_{\mathrm{k}}\mathbf{F}_{\mathrm{k}} - \mathbf{1}\right) = \frac{1}{2}(\mathbf{U}_{\mathbf{k}}^{\mathbf{2}} - \mathbf{1}) \tag{2}$$

with the three variants for the cubic$\Longrightarrow$quadratic transformation

$$\mathbf{U_1} = \begin{bmatrix} \beta & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}, \mathbf{U_2} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \alpha \end{bmatrix} \mathbf{U_3} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \beta \end{bmatrix} \tag{3}$$

with $\alpha = a/a_0, \beta = c/a_0$.
For the reorientation of the variant M_k *into the variant* M_l, the strain tensor is :

$$\mathbf{E}_{\mathrm{kl}}^{\mathrm{re}} = \frac{1}{2}(\mathbf{U}_{\mathrm{l}}^{\mathbf{2}} - \mathbf{U}_{\mathbf{k}}^{\mathbf{2}}) \tag{4}$$

Note that the interface between austenite and martensite can only exist in the form of "twinned" martensite in front of austenite .

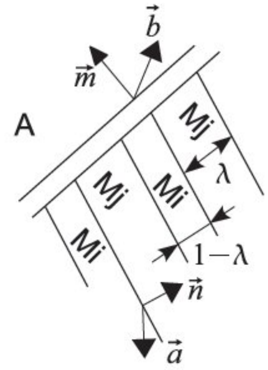


FIG.: Twinned Martensite forming an interface with austenite

In fact the CTM (Cristallographical Theory of Martensite) gives the solution of the "twinning equation"

Let us apply this to the cubic$\Longrightarrow$quadratic transformation .We shall begin with variants 1 and 2.

The calculations are as follows :

Let $\mathbf{R}$ a rotation matrix of 180° around the $\hat{\mathbf{e}}$ axis défined by :

$$\hat{\mathbf{e}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{6}$$

It is easy to verify that $\mathbf{R^T U_1 R} = \mathbf{U_2}$ and we obtain :

$$1.\ \mathbf{a} = \frac{\sqrt{\mathbf{2}}(\beta^{\mathbf{2}} - \alpha^{\mathbf{2}})}{\beta^{\mathbf{2}} + \alpha^{\mathbf{2}}} \begin{pmatrix} -\beta \\ \alpha \\ 0 \end{pmatrix}, \ \hat{\mathbf{n}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{7}$$

$$\mathbf{2}.\ \mathbf{a} = \frac{\sqrt{\mathbf{2}}(\beta^{\mathbf{2}} - \alpha^{\mathbf{2}})}{\beta^{\mathbf{2}} + \alpha^{\mathbf{2}}} \begin{pmatrix} -\beta \\ -\alpha \\ 0 \end{pmatrix}, \ \hat{\mathbf{n}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \tag{8}$$

The resolution of compatibility equation between A and $(M_1,\ M_2)$is explained in the part "martensitic transformation"
For the Ni_2-Mn-Ga $\alpha = {}^a/_{a_0}$=1.0188 et $\beta = {}^c/_{a_0}$=0.9589 one obtains $\lambda = .3083$ and
▲For $M_1 \Longrightarrow M_2$

$$\mathbf{E}^{\mathrm{re}}_{12} = \frac{1}{2}(\mathbf{U}_2^{\mathbf{2}} - \mathbf{U}_{\mathbf{1}}^{\mathbf{2}}) = diag(0.0593, -0.0593, 0) \tag{9}$$

▲For $A \Longrightarrow (M_1, M_2)$

$$\mathbf{E}^{\mathbf{tr}} = \frac{1}{2}(\mathbf{U}^{\mathbf{2}}_{\mathrm{tw}} - \mathbf{1}) \tag{10}$$

with $\mathbf{U}_{\mathrm{tw}}$=$\lambda\mathbf{U_2} + (1-\lambda)\mathbf{U}_1$and finally
$\mathbf{E}^{\mathbf{tr}} = \mathrm{diag}(-0.0224{,}0.0004, 0.0190)$

Model of magneto- thermo-mechanical behavior of MSMA single crystal

Gibbs free energy (Thermodynamic potential chosen)

$$
\begin{gathered}
G(\boldsymbol{\Sigma}, \boldsymbol{T}, \mathbf{h}, z_0, z_1 \ldots z_{n,}, \alpha, \theta, \alpha_A) = G_{chem}(T, z_0) + G_{therm}(T) \\
+ G_{meca}(\boldsymbol{\Sigma}, z_0, z_1 \ldots z_{n,}) + G_{mag}(\boldsymbol{T}, \mathbf{h}, z_0, z_1 \ldots z_{n,}, \alpha, \theta, \alpha_A)
\end{gathered}
\tag{11}
$$

$$
\sum_{k=0}^{k=3} z_k = 1 \tag{12}
$$

$\boldsymbol{\Sigma}$ stress tensor , $\mathbf{h} = \mathrm{H}\boldsymbol{x}$applied magnetic field, T temperature z_0 austenite fraction z_k variant M_k fraction , α Weiss domain proportion within the REV of a martensite variant ; α_A Weiss domain proportion of A.

REV

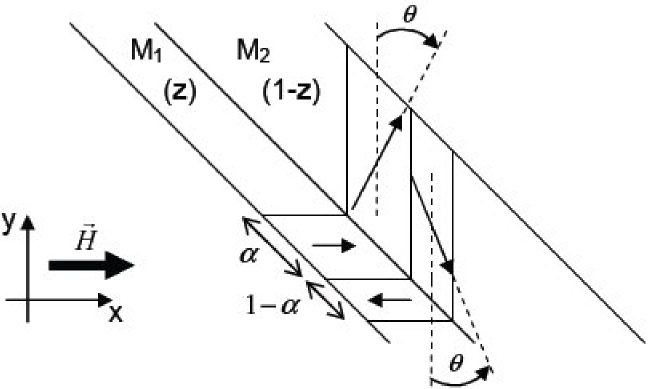


FIG.: Representative Elementary Volume of two variants M_1 and M_2 ($z_1 = z$, $z_2 = 1 - z$).

REV under magnetic field H

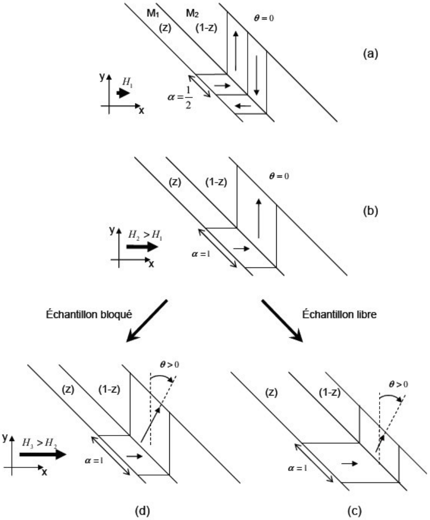


FIG.: Evolution of the Representative Elementary Volume under the influence of a magnetic field .

Chemical energy expression

This energy relates the latent heat associated with an $A \Longrightarrow M$. phase transformation :

$$G_{chem}(T, z_0) = \left(u_0^A - Ts_0^A\right) z_0 + \left(u_0^M - Ts_0^M\right)(1 - z_0) = u_0^M - Ts_0^M + \Pi_0^f(T) \tag{13}$$

with $\Pi_0^f(T) = \triangle u - T\triangle s$
and $\triangle u = u_0^A - u_0^M ; \triangle s = s_0^A - s_0^M$
this formulation is the same than classical SMAs.

Thermal energy expression

Under the specific heats are the same for austenite and martensite , with the definition:

$$C_p = -T\frac{d^2 G_{therm}}{dT^2} \tag{14}$$

The thermal energy is obtained under double integration :

$$G_{therm} = C_p\left[(T - T_0) - TLn\left(\frac{T}{T_0}\right)\right] \tag{15}$$

Mechanical energy expression

For a single crystal made of the mother phase A and n martensite variants, the G_{mech} expression can be chosen as :

$$\rho G(\boldsymbol{\Sigma}, T, z_0, z_1, ..., z_n) = -\boldsymbol{\Sigma} : \sum_{k=0}^{k=3} z_k \mathbf{E}_{\mathbf{k}}^{\mathbf{tr}} - \frac{1}{2} \boldsymbol{\Sigma} : \boldsymbol{M} \boldsymbol{\Sigma} + \phi_{it}(z_0, z_{1,} ... z_n) \quad (16)$$

.

with:

$$\phi_{it} = A z_0 (1 - z_0) + \frac{1}{2} \sum_{k=1}^{n} \sum_{l=1}^{n} H_{kl} z_k z_l \quad (17)$$

with l différent from k and z global martensite fraction :

$$z = \sum_{k=1}^{n} z_k = 1 - z_0 \quad (18)$$

In the following, we shall limit ourselves to the case where $\mathbf{h} = \mathrm{H}\boldsymbol{x}$et and an uniaxial compression in the direction $\mathbf{y}$

$$\boldsymbol{\Sigma} = diag\,(0, \sigma, 0) \quad (19)$$

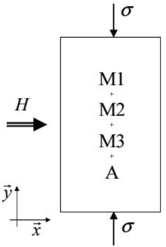


Figure: Magneto-mechanical sollicitation of a Ni-Mn-Ga single crystal (phase transformation cubic-quadratic).

In this simple case ,the mechanical energy expression is reduced to:

$$\rho G_{meca}(\sigma, z_0, z_1, z_2 z_3) = -\frac{\sigma}{2}\left[(z_1+z_3)(\alpha^2-1)+z_2(\beta^2-1)\right] \\ -\frac{1}{2}\frac{\sigma^2}{E^\star}+Az_0(1-z_0)+K(z_1z_2+z_2z_3+z_3z_1) \tag{20}$$

with $E^\bigstar$ the Young modulus and considering that the interactions between the martensite variants have the same weight.
In addition ,a restriction is to consider:

$$\sum_{k=0}^{k=3} z_k = 1 \tag{21}$$

This means that among the four volume fractions, only three are independent.

Magnetic energy expression

The expression of $\rho\, G_{mag}(H)$ will be as follows :

$$\rho\, G_{mag}(H) = -\int_0^H \mu_0 m dH \tag{22}$$

We shall use the notations m_1, m_2 m_3 the magnetizations of the three martensite variants M_1,M_2 M_3 and m_0 the magnetisation of l'austenite
Magnetization of martensite :axis of easy aimantation :

$$m_1(H) = m_s(2\alpha(H)\text{-}1) \tag{23}$$

where m_s*is* the saturation magnetization $\alpha \in [0,1]$represents the proportion of the Weiss domain.
Thus α is chosen as a linear function of H

$$(2\alpha(H)\text{-}1) = \frac{\chi_a H}{m_s} \tag{24}$$

with $m_1(H) = \chi_a H$.

Magnetization of martensite :axis of difficult aimantation.
Magnetization along the axis of difficult aimantation is chosen as corresponding to a rotation of the magnetization within the variant in question. On the basis of the REV choice made above :

$$m_2(H) = m_3(H) = m_s sin(\theta\,(H)\,) \tag{25}$$

where $\theta \in \left[-\frac{\pi}{2}\,,\,\frac{\pi}{2}\right]$represents the angle of rotation of the magnetization.
We shall choose $sin(\theta\,(H)\,)$ as linear in H in the form :

$$sin(\theta\,(H)\,) = \frac{\chi_t H}{m_s} \tag{26}$$

This is to say m_2=m_3=χ_tH.

Magnetization of austenite

With an operational temperature lower than the Curie temperature of the material,the behavior is considered to be similar to that of variant M_1

$$m_0(H) = m_s(2\alpha_A(H)\text{-}1) \tag{27}$$

where $\alpha_A \in [0\,,1\,]$represents the proportion of the Weiss domain in the austenite. α_A is chosen as a linear function of H

$$(2\alpha_A(H)\text{-}1) = \frac{\chi_A H}{m_s} \tag{28}$$

Mixing rule

The mixture rule then gives the global magnetization of the material :

$$m(H) = \sum_{k=0}^{k=3} z_k M_k \tag{29}$$

$$m(H) = m_s \left(z_0(2\alpha_A(H)\text{-}1) + z_1(2\alpha(H)\text{-}1) + (z_2 + z_3) sin(\theta\,(H)\,)\right) \tag{30}$$

The curves given by Likhachev et al show that when z=1 ; m=M is linear in H_0=H with slope χ_t and for z=0 ;m linear in H with slope χ_a.
In this two-variants model (z_3=0), z_1=z ; z_2=1-z) we write :

$$m = m_x = \chi_a Hz + \chi_t H(1-z) \tag{31}$$

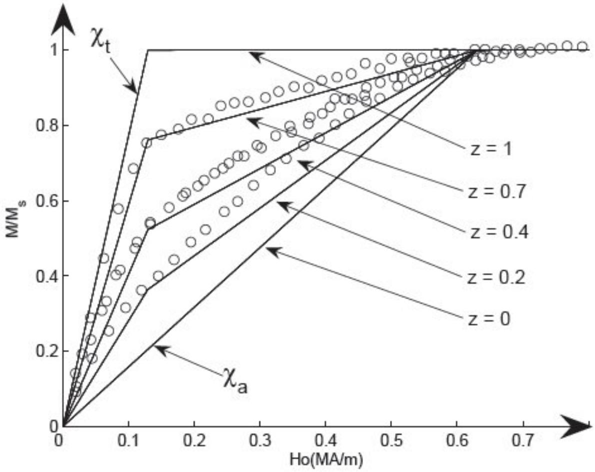


FIG.: Magnetization curves for different fractions z of variant 1(two variants model ; 1 and 2) model :lines ; experiments (*o*)(Experiments Likhachev et al.).

As it was performed in Gauthier thesis (2007) , the G_{mag} expression is established :

$$\rho\, G_{mag}(H, z_0, z_1, z_2, z_3, \alpha, \theta, \alpha_A) = \tag{32}$$

$$-\mu_0 m_s \left[z_1((2\alpha-1)\,H - \frac{m_s}{2\chi_a}(2\alpha-1)^2) + (z_2+z_3)\left(sin(\theta)\,H - \frac{m_s}{2\chi_t} sin^2(\theta) \right) \right. \tag{33}$$

$$\left. -\mu_0 m_s \left[z_0((2\alpha_A-1)\,H - \frac{m_s}{2\chi_A}(2\alpha_A-1)^2) \right] \right. \tag{34}$$

The experimental curves observation shows that m_s is not constant but rather depends on the temperature .For ferromagnetic materials the Weiss theory gives the dependence of m_s *with* T by an implicit equation delivered by Zuo et al.

$$\frac{m_s(T)}{m_s^0} = tanh\left(\frac{m_s(T)}{m_s^0} \frac{T_c}{T} \right) \tag{35}$$

where T_c is the Curie temperature m_s^0 the magnetization at 0°K.In order to simplify the calculations, the parameters m_s^0 *and* $T_c$will be taken to be identical for the austenite and the martensite although in reality they are slighty different.

Free energy expression

For a single crystal with an austenitic phase and three martensite variants for a transformation cubic $A \Longrightarrow$quadratic M_i) under a thermo-magneto-mechanic loading ,the Gibbs free energy expression can be written as

$$
\begin{aligned}
&\rho G(H, \sigma, T, z_0, z_1, z_2, z_3, \alpha, \theta, \alpha_A) = \\
&u_o^M - T s_o^M + z_o(\Delta U - T \Delta S) \\
&+ C_p \left[(T - T_o) - T \cdot \ln \left(\frac{T}{T_o} \right) \right] \\
&- \frac{\sigma}{2} \left((z_1 + z_3) \left(\beta_a^2 - 1 \right) + z_2 \left(\beta_c^2 - 1 \right) \right) \\
&- \frac{1}{2} \frac{\sigma^2}{E} + A z_o (1 - z_o) + K (z_1 z_2 + z_1 z_3 + z_2 z_3) \\
&- \mu_0 m_s(T) \left[z_1 \left((2\alpha - 1) H - \frac{m_s(T)}{2\chi_a} (2\alpha - 1)^2 \right) \right. \\
&+ (z_2 + z_3) \left(\sin(\theta) H - \frac{m_s(T)}{2\chi_t} (\sin(\theta))^2 \right) \\
&\left. + z_0 \left((2\alpha_A - 1) H - \frac{m_s(T)}{2\chi_A} (2\alpha_A - 1)^2 \right) \right]
\end{aligned}
$$

with $\sum_{k=0}^{3} z_k = 1$

with $\beta_a = \alpha$ *et* $\beta_c = \beta$
This G expression is a little complicated but can be subdivised into a number of specific situations(purely magnetic or mechanical or thermal loading).

Clausius-Duhem inequality

Thermodynamic forces

$$\blacktriangle\; E = -\rho \frac{\partial G}{\partial \sigma} = \frac{\sigma}{E\star} + \frac{1}{2}\left[(z_1+z_3)(\alpha^2-1) + z_2(\beta^2-1)\right] \quad (36)$$

$$\blacktriangle\; \mu_0 m = -\rho \frac{\partial G}{\partial H} = \mu_0 m_s \left[z_0(2\alpha_A - 1) + z_1(2\alpha-1) + (z_2+z_3) sin(\theta)\right] \quad (37)$$

$$\bullet\; \rho s = -\frac{\partial \rho G}{\partial T} = s_o^M + z_o \Delta S + C_p \ln\left(\frac{T}{T_o}\right)$$
$$+ \mu_0 \frac{dm_s}{dT} H \left[z_1(2\alpha-1) + (z_2+z_3)\sin(\theta) + z_0(2\alpha_A - 1)\right]$$
$$-2\mu_0 m_s(T) \frac{dm_s}{dT}\left[z_1 \frac{(2\alpha-1)^2}{2\chi_a} + (z_2+z_3)\frac{\sin^2(\theta)}{2\chi_t} + z_0 \frac{(2\alpha_A-1)^2}{2\chi_A}\right]$$

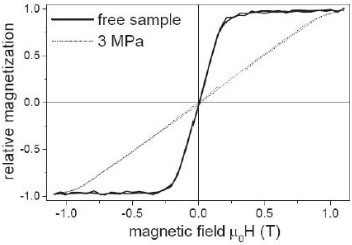
A magneto-thermal effect is present in the entropy expression due to the dependence with temperature of m_s .

The thermodynamical forces associated to variables α , α_A et θ are taken equal to zero,i.e.

$$\rho \frac{\partial G}{\partial \alpha} = 0 \, , \, \rho \frac{\partial G}{\partial \alpha_A} = 0 \, , \, \rho \frac{\partial G}{\partial \theta} = 0 \tag{38}$$

The free energy expression choice confirms that purely magnetic behavior is considered to be reversible.

Effectively , the magnetization curves measured by Heczko et col on two samples (one at stress free state and the other under 3 MPa) don't exhibit hysteresis.



Finally, the thermodynamic forces associated at the austenite fraction and martensite variants ones are writtten

$$\blacktriangle\, \pi_0^f = -\rho \frac{\partial G}{\partial z_0} = \begin{array}{c} -\triangle U - T\triangle S - A(1-2z_0) \\ +\mu_0 m_s \left[(2\alpha_A - 1) H - \frac{m_s}{2\chi_A} (2\alpha_A - 1)^2 \right] \end{array} \tag{39}$$

$$\blacktriangle\, \pi_1^f = -\rho \frac{\partial G}{\partial z_1} = \begin{array}{c} \frac{\sigma}{2} \left(\alpha^2 - 1\right) - K\left(z_2 + z_3\right) \\ +\mu_0 m_s(T) \left[((2\alpha - 1) H - \frac{m_s}{2\chi_a} (2\alpha - 1)^2) \right] \end{array} \tag{40}$$

$$\blacktriangle\, \pi_2^f = -\rho \frac{\partial G}{\partial z_2} = \begin{array}{c} \frac{\sigma}{2} \left(\beta^2 - 1\right) - K\left(z_1 + z_3\right) \\ +\mu_0 m_s(T) \left(sin(\theta) H - \frac{m_s}{2\chi_t} sin^2(\theta) \right) \end{array} \tag{41}$$

$$\blacktriangle\, \pi_3^f = -\rho \frac{\partial G}{\partial z_3} = \begin{array}{c} \frac{\sigma}{2} \left(\alpha^2 - 1\right) - K\left(z_1 + z_2\right) \\ +\mu_0 m_s(T) \left(sin(\theta) H - \frac{m_s}{2\chi_t} sin^2(\theta) \right) \end{array} \tag{42}$$

The behavior is irreversible , so the Clausius-Duhem inequality can be written as :

$$dD = -\rho dG - \mu_0 m dH - \varepsilon d\sigma - s dT \geq 0 \tag{43}$$

where dD constitutes the dissipation increment . Its expression can be written as :

$$dD = \sum_{i=0}^{3} \pi_i^f dz_i \geq 0 \; avec \; \sum_{i=0}^{3} dz_i = 1 \tag{44}$$

Kinetics of phase transformation or reorientation

Example of 2 martensite variants :

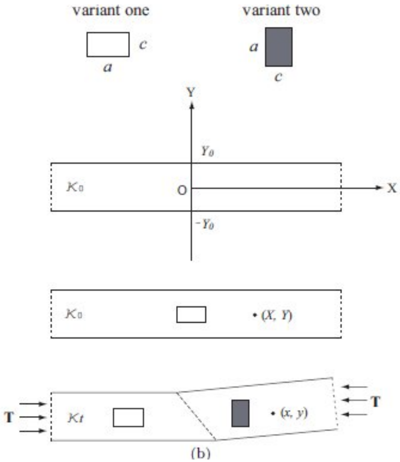


FIG.: Representation of a 2D network of two martensite variants induced by an uniaxial compression.

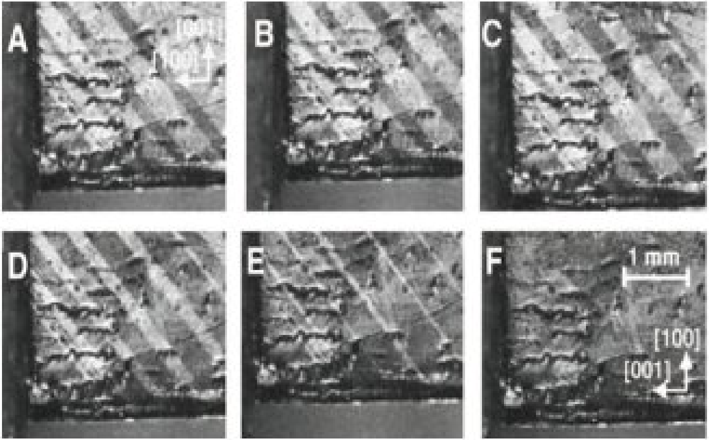


FIG.: Morphology of surface for a Ni-Mn-Ga plate in white :the original variant ; in black : the variant created by uniaxial compression)

If a sample contains two martensite variants M_1 *and* M_2,during the thermal evolution. Let $z = z_1 = 1 - z_2$ and the Clausius -Duhem inequality becomes

$$dD = \pi_1^f dz_1 + \pi_2^f dz_2 \geq 0 \tag{45}$$

$$dD = (\pi_1^f - \pi_2^f) dz \geq 0 \tag{46}$$

The reorientation begins when $(\pi_1^f - \pi_2^f) \geq \pi_{cr}(\mathrm{T})$ for the path (a) and when $(\pi_1^f - \pi_2^f) \leq -\pi_{cr}(\mathrm{T})$ for the path (b). After the linitiation of the reorientation, the behavior is modeled with the following kinetic :

$$\dot{\pi_1}^f - \dot{\pi_2}^f = \lambda \dot{z} \; avec \; \dot{z} = \dot{z_1} = -\dot{z_2} \tag{47}$$

One can take λ as a constant or the λ value can be taken as dependent of the anterior deformation . Hence, the concept of the memory point is introduced and one makes the distinction between internal and external loops.

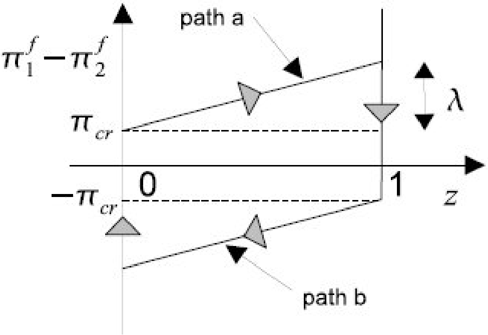


FIG.: Thermodynamical force as function martensite fraction z_1*of* M_1.

For π_{cr}(T) function of temperature,a linear dependence is chosen

$$\pi_{cr}(T) = \pi_{cr}^{0} + k_{cr}\left(A_{s}^{0} - T\right) \tag{48}$$

Equivalence between the actions of the magnetic field H and the stress σ
De façon classically, reorientation occurs when the thermodynamic force $\pi^{f\bigstar}$reaches the values π_{cr}.In the two variants model M_1 and M_2

$$\pi^{f\bigstar}(\sigma, H, z = 0) = \pi_{cr} \tag{49}$$

with

$$\pi_{cr} = \sigma\gamma - K_{12} - \mu_0 m_s^2 \left(\frac{(1-2\alpha)\, sin\theta}{\chi_t} + \frac{(2\alpha-1)^2}{2\chi_a} + \frac{sin^2\theta}{2\chi_t}\right) \tag{50}$$

Three situations must be examined

▲ Zone I : no saturation in α *et* θ

▲ZoneII : saturation in α , *not in* θ

▲Zone III : saturation in α *et* θ

The figure allows the comparison between the measured values and the predictions with :

$\mu_0 m_s$=0.65 T, χ_t=0.82, χ_a=4, $\pi_{cr}+K_{12}$=2.10^{4}Pa, $\gamma = 0.055$

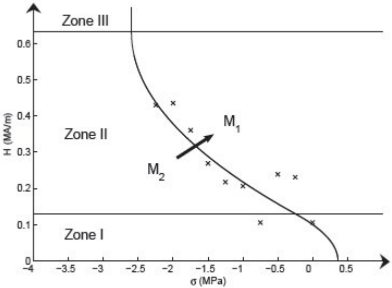


FIG.: Borderline between σ and H for the initiation of reorientation $M_2 \Longrightarrow M_1$.solid line : simulation ;(x) experimental points .

Generalization to the three martensite variants and the austenitic phase .
We generalize the concept of critical force $\pi_{cr}(T)$ and kinetics to the three martensite variants and to the austenitic phase. The figure represents the phase state and the kinetics associated . c_{ij} represents the transformation rate from M_i to M_j and $c_{0j}$from A to M_j

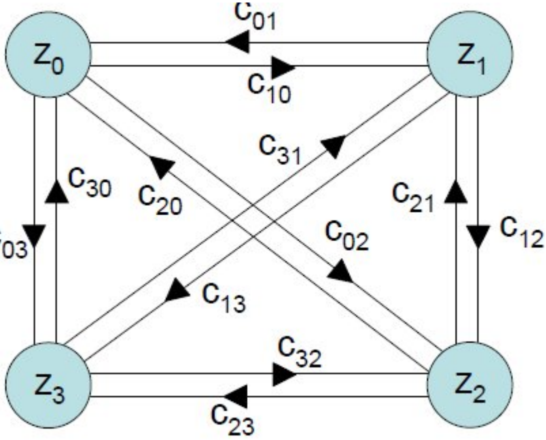


FIG.: Schematic representation of the kinetics.

and the following relations are verified

$$\dot{z}_0 = c_{10} + c_{20} + c_{30} - c_{01} - c_{02} - c_{03} \tag{51}$$

$$\dot{z}_1 = c_{01} + c_{21} + c_{31} - c_{10} - c_{12} - c_{13} \tag{52}$$

$$\dot{z}_2 = c_{02} + c_{12} + c_{32} - c_{20} - c_{21} - c_{23} \tag{53}$$

$$\dot{z}_3 = c_{03} + c_{13} + c_{23} - c_{30} - c_{31} - c_{32} \tag{54}$$

with the c_{ij} défined by
$$\begin{gathered} c_{ij} = 0 \textit{ if } \pi_j^f - \pi_i^f \leq \pi_{cr}(T) \textit{ or } z_i = 0 \\ c_{ij} = \tfrac{1}{\lambda}\left(\dot{\pi}_j^f - \dot{\pi}_i^f\right) \textit{ otherwise} \end{gathered}$$

Different values of λ have to take into account : λ_A for the transformation $A \Rightarrow M_i$ and λ_M for $M_i \Rightarrow M_j$.

Identification of parameters

The material parameters are strongly dependent of the alloy composition . Its identification need specific measurements as for example the X-ray measurements for lattice parametersc, the DSC (differential scanning calorimétry) for phase transformation temperatures. measurements of susceptibitly , Curie temperature and last compression tests in order to know the Young modulus and hardening
"Differential scanning calorimetry"
La DSC gives us . the four phase transformation temperatures (at stress free state) : M_f^0 M_s^0 ,A_s^0 , A_f^0.To begin with, the hysteresis curve area is equal to $-\triangle U$..As the model show by verifying $A_f^0 - A_s^0 \simeq M_s^0 - M_f^0$ one can obtain :

$$\triangle S = \frac{2\triangle U}{A_s^0 + M_s^0} \tag{55}$$

$$A = \frac{-\triangle S\left(A_s^0 - M_s^0\right)}{2} \tag{56}$$

$$\lambda_A = -\triangle S(A_f^0 - M_s^0) \tag{57}$$

Crystallographic measurements :

The lattice parameters a_0 , a *et* c are obtained with X-rays

Magnetic measurements :

The curves of m as a function of H for different temperatures may serve to identify T_c , m_s^0 χ_a χ_t χ_A.

Mechanical measurements :

The curves of reorientation at different temperatures lower than A_s^0, can be taken in order to obtain $\pi_{cr}(T)$, λ_M *et* $E^\star$.

The selected parameters are reported in table 1

$A_S^o = 309.4$ K	$M_S^o = 301.7$ K	$A = 5.48.10^5$ J/m^3
$a_o = 5.84$ Å	$a = 5.95$ Å	$c = 5.60$ Å
$E = 5.10^9$ Pa	$\lambda_M = 4.10^5$	$K = 0$
$\chi_a = 5$	$\chi_t = 1.05$	$\chi_A = 1.76$
$T_c = 370$ K	$m_{S0} = 710$ kA/m	$\lambda_A = 1.26.10^6$ J/m^3
$\pi_{cr}^o = 12.10^3$ J/m^3	$k_{cr} = 800$ Pa/K	

Reliability of the model

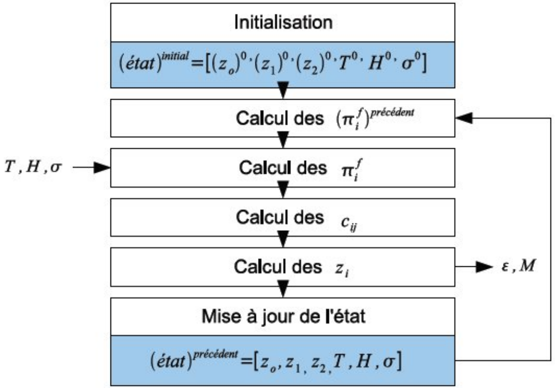


FIG.: Algorithm for simulating the general behavior of an MSMA.

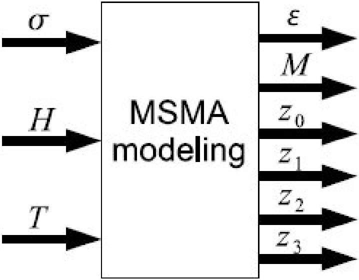


FIG.: Inputs and outputs from the digital simulation.

La figure presents the l'evolution of the déformation $\varepsilon$and volume fractions of variants z_i with the température T. The deformation is around 2% when the magnetic field is applied, and -4% under the influence of compressive stress. Naturally, there is not deformation in the absence of stress and magnetic field and at low temperature the fractions z_i $i = 1, 2, 3$ are equal (at 1/3).

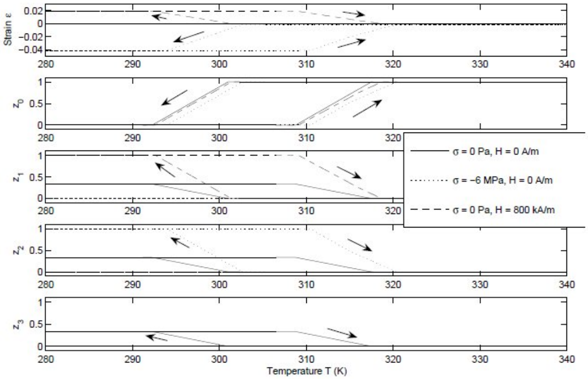


FIG.: Results of our simulation of a thermal action with and without magnetic field or stress.

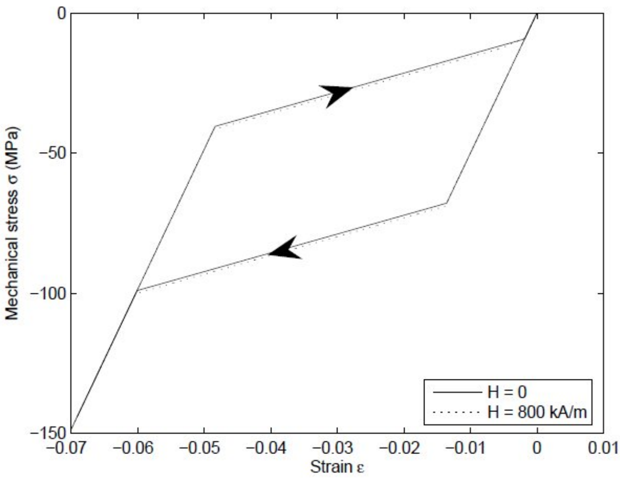


FIG.: Simulation of the mechanical action at high temperature (pseudo-elasticity) :T=320K

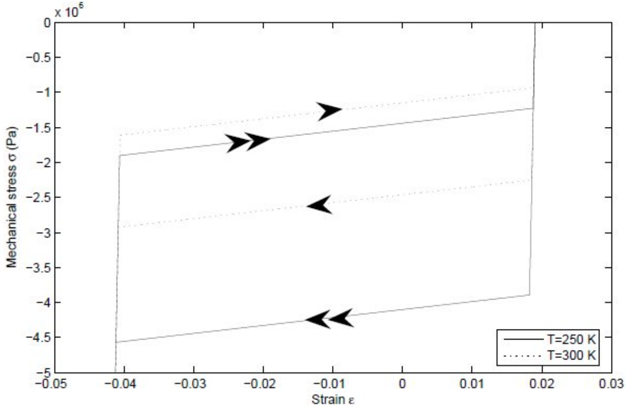


FIG.: Mechanical action at low temperature giving rise to the martensite reorientation under $H = 800 kA/m$.

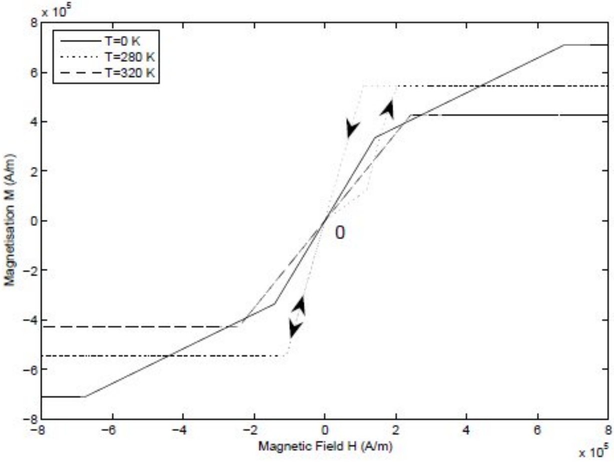


FIG.: Magnetization curves for different isotherms .

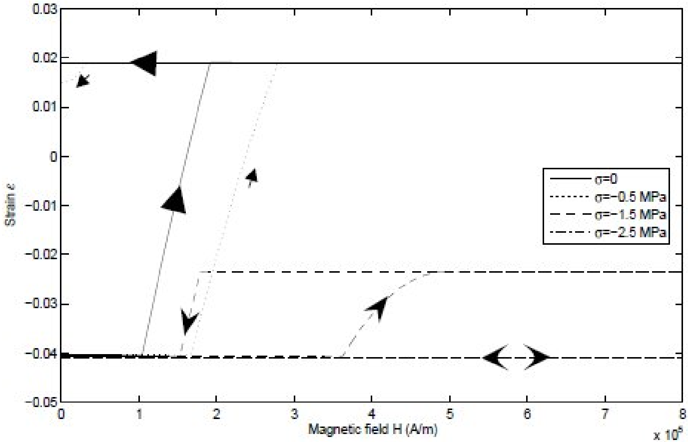


FIG.: Evolution of the deformation ε with the magnetic field H for different levels of applied stress.

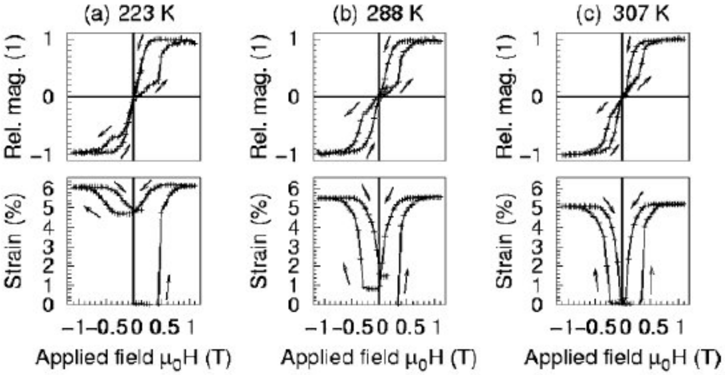


FIG.: Evolution of the deformation and magnetization under a fixed stress $\sigma = -1$MPa.Experiments performed by Straka et al..

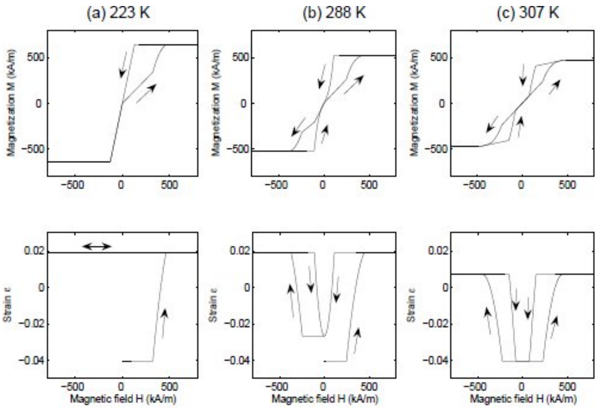


FIG.: .Evolution of the deformation and magnetization under a fixed stress $\sigma = -1$MPa.Modeling of experiments performed by Straka et al.

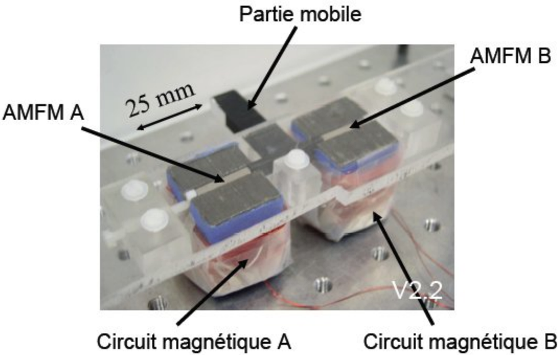


Fig.: Photograph of the “ Push-Pull” actuator.

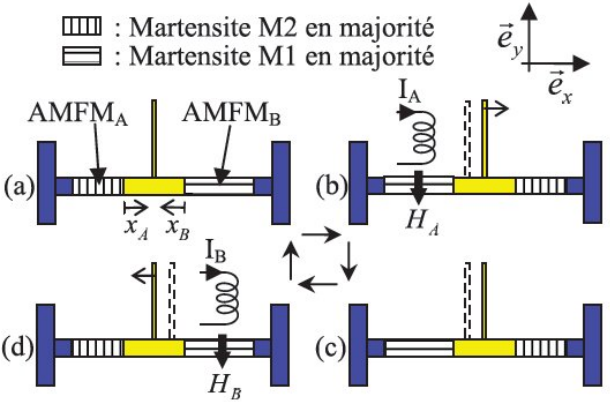


FIG.: Principle of operation of Push-Pull actuator.

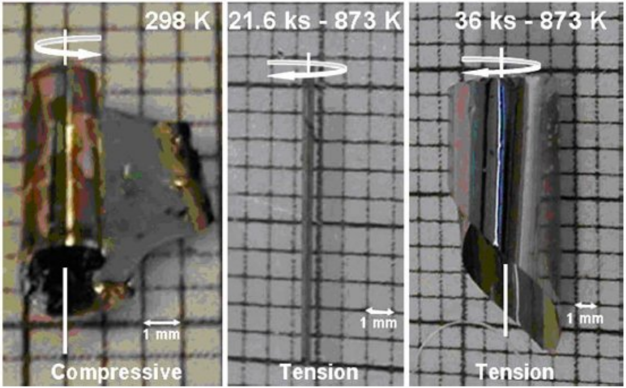


FIG.: Photograph of three films deposited at 298 °K and annealed during 21.6 ks and 36 ks at 873 K respectively. (The arrows indicate the directions of rolling).